

Concrete Module

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Info



Concrete Module 6

Copyright:

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110930

Latest information about programs from **WIN-Statik** and **FEM-Design** see

www.strusoft.com

1 Introduction

1.1 Generally

With the program **Concrete Module** continuous beams and plane frame structures can be designed according to the active concrete code. The **Concrete Module** is used together with the program **Frame Analysis** in which calculations of section forces are performed.

Unlike beams and columns made of steel and wood, beams and columns made of concrete have been composed of more than one material. Extended input data is therefore necessary for concrete and concrete and reinforcement.

Concrete material properties are furthermore by nature such that cracking is normal, which results in that the structure stiffness is being altered when it is exposed to a load. This condition is being mastered with an iterative method of calculation.

Design with consideration taken to buckling in the frame plane can be performed for second order section forces according to the *General method* [1] 5.8.6 or for first order theory with help from buckling lengths according to the *Nominal stiffness method* [1] 5.8.7 or the *Nominal curvature method* [1] 5.8.8.

Continuous support, as well as support in certain points, of the beam can be defined.

The program shows all code prescribed checks depending on type of section and current load.

1.2 Restrictions

The support conditions in the shape of joints/hinged, fixed or cantilever can be defined with consideration to buckling and lateral instability.

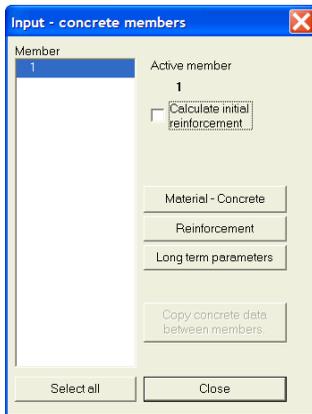
Important! A joint that has been defined with the **Member** tool will automatically be considered supported with respect to bending instability out of the frame plane at a possible design. You can use the **Unbraced joint** tool if you don't want support against buckling, as the joint is only considered to be sufficiently torsion stiff to be assumed as a fork support with consideration to lateral instability. Loads that affect the construction are assumed to be working through the shear centre **SC** of the cross section, which means that there is no torsion of the cross section.

Controls of the support capacity with consideration to lateral instability are not performed at the moment.

2 Program structure

2.1 Concrete input

The extended input data for concrete and reinforcement concerns material properties as well as reinforcement detailing rules. There are moreover loadcase related properties.



The various input data options, which can be made for each concrete member in the frame, are displayed in the picture beside.

Design of initial reinforcement

Check box for options whether design of initial reinforcement is to be performed or not.

When design of the initial reinforcement has been selected the reinforcement input is limited to material data and measurement data (cover of reinforcement and so on) that control the distribution of reinforcement (see ch. 2.1.1 - 2.1.2).

During the calculation of initial reinforcement a simplified frame analysis is being performed without consideration to cracking or reinforcement since this option can only be selected after all members have been provided with reinforcement. This results in that the automatically calculated reinforcement can only be seen as an approximate first suggestion.

When calculation of the initial reinforcement has been performed the check box is automatically disabled. The user can also disable the check box.

2.1.1 Material - Concrete

The material input is separated in three tabs, **Material**, **Reinforcement details**, and **Calculation settings**.

2.1.1.1 Material data

In the material dialog box all material strength input is assembled. Note that individual settings can be made for each frame-member. By pressing one of the buttons **M** a dialog box for showing the complete data of the selected material or for defining a user defined material is opened.

The screenshot shows a software dialog box titled "Input - Member: 1". It has tabs for "Material", "Reinforcement details", and "Calculation settings". The "Material" tab is active. Under the "General" section, there are dropdown menus for "Exposure class" (set to "X0 Very dry") and "Life class" (set to "L50"). There are two unchecked checkboxes: "Quality control and reduced deviations" and "Reduced or measured geometrical data". To the right, under "Concrete (MPa)", there is a "Strength class" dropdown (set to "C30/37") and a button "M". Below this is an unchecked checkbox "Low strength variation (< 10 %)". Further down are input fields for f_{cd} (20.00), f_{ctd} (1.35), and E_{0d} (27364). The "Design values Reinforcement (MPa)" section has a "Bottom" tab selected. It contains a table with columns for "Bottom", "Top", "Stirrups", and "Longitudinal". Each column has a "Designation" dropdown (all set to "B500") and a button "M". Below the designations are rows for "Diameter, mm", f_{yd} , f_{ycd} , and E_{sd} .

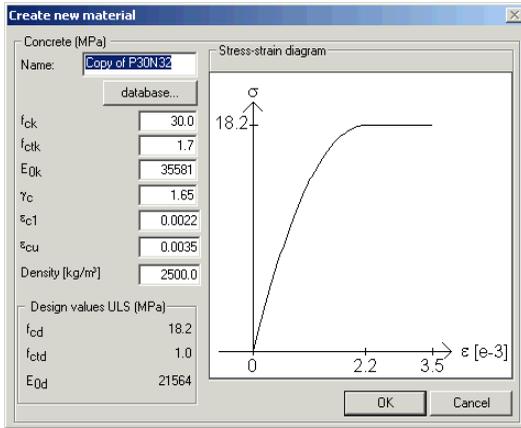
| | Bottom | Top | Stirrups | Longitudinal |
|--------------|--------|--------|----------|--------------|
| Designation: | B500 | B500 | B500 | B500 |
| Diameter, mm | 20 | 20 | 8 | 8 |
| f_{yd} | 435 | 435 | 435 | 435 |
| f_{ycd} | 435 | 435 | 435 | 435 |
| E_{sd} | 200000 | 200000 | 200000 | 200000 |

General

Under the header **General**, conditions that affect the calculation are stated. Exposure class and life time class are defined and affect code checks where relevant. The options **Quality control and reduced deviation** and **Reduced or measured geometrical data** can be checked. The two latter options together with the option **Low strength variation**, see below, changes the partial factors for materials according to EN 1992-1-1 Annex A A2.1 and A2.2.

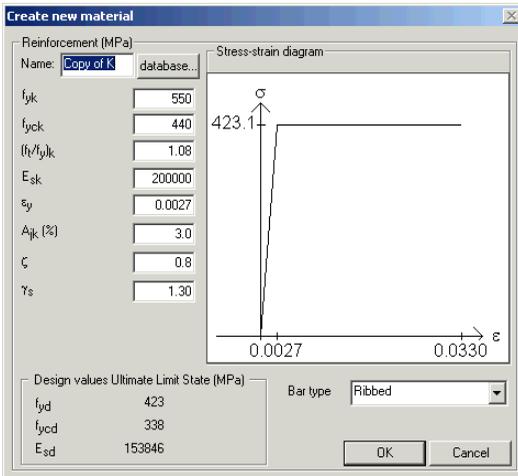
Concrete

Concrete strength class can be defined by selecting from the list. Current design values are shown. If the option **Low strength variation** is checked together with one or both of the two options described above the partial factor for concrete will be changed according to EN 1992-1-1 Annex A A2.1 and A2.2.



By pressing button **M** beside the strength class the complete current data is being shown, as is a graphic presentation of the stress-strain diagram. The values in the material data are available for changes when the option **Create new** is chosen.

Reinforcement



In the **Reinforcement** dialog boxes type of reinforcement for bottom- and top bars and stirrups as well as for longitudinal bars can be defined by selecting from lists. Longitudinal bars are only used in the program **Concrete Section** to resist torsion and biaxial shear. Current design values as well as current diameters are shown.

By pressing button **M** beside the strength class the complete current data is shown, as is a graphic presentation of the stress-strain diagram. The values in the material data are available for changes when the option **Create new** is chosen.

For the British Annex also the **W/c** ratio is defined depending on current exposure class. This option affects the required cover according to Table NA.2 and NA.3 in the British Annex.

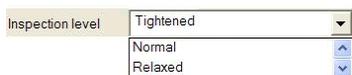
Danish Annex

In the Danish Annex environmental classes are defined instead of exposure classes. The relations between these are the following:

Table 2426-1 – Normative assignment of exposure classes to environmental classes:

| Environmental class | Passive | Moderate | Aggressive | Extra aggressive |
|--|---------|----------|------------|------------------|
| Covers the following exposure classes according to DS/EN 206-1 | X0 | XC2 | XD1 | XD2 |
| | XC1 | XC3 | XS1 | XD3 |
| | | XC4 | XS2 | XS3 |
| | | XF1 | XF2 | XF4 |
| | | XA1 | XF3 | XA3 |
| | | | XA2 | |
| | | | | |
| | | | | |

For the Danish Annex also the **Inspection Level** should be defined which affects the partial factor for materials.



If the option **Prefabricated elements** is checked the partial factors for material is changed.

The options **Reduced or measured geometrical data**, **Reduced or measured geometrical data** and **Low strength variation** are not used in the Danish Annex.

Finish Annex

For the Finish Annex also the **Construction class** are defined which affects the required cover.

2.1.1.2 Reinforcement details

In the **Reinforcement details** dialog box bar diameters, cover of reinforcement, min spacing between bars and so on used to distribute calculated amounts of reinforcement can be defined.

The screenshot shows a software dialog box titled "Input - Member: 1" with three tabs: "Material", "Reinforcement details", and "Calculation settings". The "Reinforcement details" tab is active and contains the following fields:

| Main bar details | | Bottom | Code | Top | Code |
|--------------------------|--|--------|------|-----|------|
| Diameter (mm) | | 20 | | 20 | |
| Cover (mm) | | 30 | 30 | 30 | 30 |
| Cover side (mm) | | 30 | 30 | | |
| Distance between bars: | | | | | |
| In same layer (mm) | | 20 | 20 | 20 | 20 |
| In different layers (mm) | | 20 | 20 | 20 | 20 |
| Vibration space (mm) | | 0 | | | |
| Largest aggregate size | | 10 | | | |
| Cover deviation (mm) | | 10 | | | |

Additional options in the dialog include:

- Use regular spacing in layer
- Code control
- Stirrup details**
 - Use stirrups
 - Diameter (mm): 8
 - Min. s dist (mm): 30
 - Min stirrups on whole length

Buttons at the bottom right: OK, Cancel.

Main bar details

The diameter for bottom and top reinforcement are defined. If standard reinforcement is chosen the diameters are the ones available for that type. For non standard reinforcement the available diameters are 1-99 mm.

Further on the covers at bottom, top and side are defined together with allowed distances between bars in the same layer and for different layers.

How the reinforcement is positioned is also influenced by the options **Vibration space** and **Largest aggregate size**.

The option **Cover deviation** represents the part of the total cover called Δc_{dev} according to EN 1992-1-1 4.4.1.3.

The program will choose minimum values (displayed with blue print) for the current diameter and code with the **Code value** button.

When the **Code control** option of input data has been activated the chosen values will be controlled, and compared with, existing rules for the current code.

Stirrup details

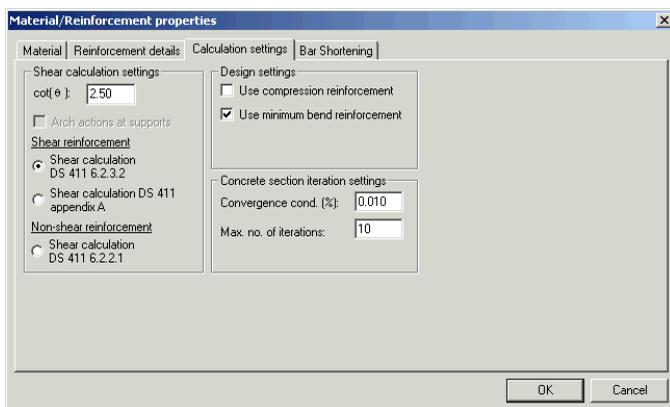
The user can choose to calculate completely without stirrups and will then get a warning if the shear capacity is not adequate. The calculation will then be performed according to EN 1992-1-1 6.2.2.

If stirrups are used the diameter is chosen, the angle between stirrup and beam axis and the minimum allowed distance between the stirrups.

Finally the user defines if minimum shear reinforcement according to EN 1992-1-1 9.3.2 should be considered. If activated this option will also be considered if calculation without shear reinforcement is chosen, see below.

2.1.1.3 Calculation settings

In the **Calculation settings** dialog different options governing the calculation can be set.



Shear calculation settings

Cot θ is defined where θ is the angle between the concrete compression strut and the beam axis.

The value affects the amount of required shear reinforcement according to EN 1992-1-1 6.2.3 and:

$$1,0 \leq \cot \theta \leq 2,5$$

The user can choose between shear calculation with shear reinforcement according to EN 1992-1-1 6.2.3 and without shear reinforcement where this is allowed according to EN 1992-1-1 6.2.2.

For the latter option it is also possible to choose **Enhanced shear strength near supports**.

2nd order moment calculation

Design for flexural buckling with regard to 1th order theory can be performed according to [1] 5.8.7 *Method based on nominal stiffness* or [1] 5.8.8 *Method based on nominal curvature*.

Design settings

The user can define if compression reinforcement, if needed should be used or not.

The user can define if minimum bending reinforcement according to EN 1992-1-1 9.2.1.1 or 9.3.1.1 should be considered.

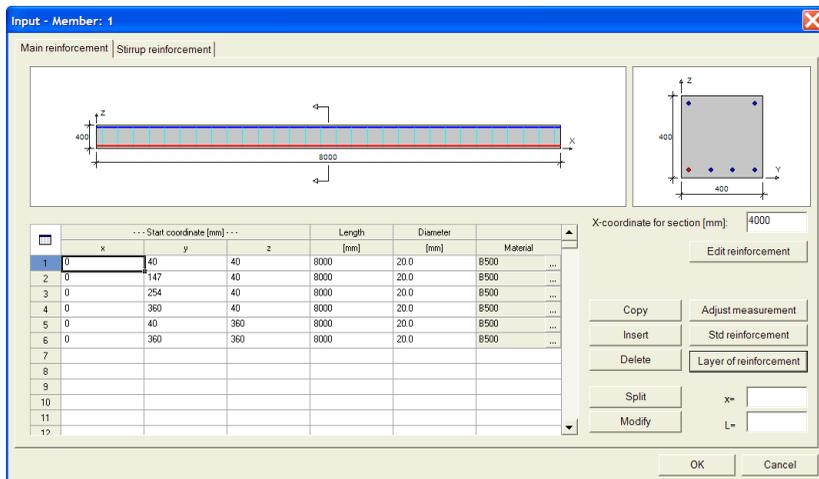
The user can define if the special rules for monolithic constructions according to EN 1992-1-1 9.2.1.2 should be considered.

Concrete section iteration settings

Input for the iterative section calculation can here be adjusted if needed.

2.1.2 Reinforcement

2.1.2.1 Main reinforcement



Under the **Main reinforcement** tab you can see reinforcement in the upper- and lower edges tabulated as well as graphically. An optional section can be displayed by (continuously) dragging the section arrow with the mouse pointer, or by giving a numerical **X**-coordinate value for the position of the section. By pointing at a reinforcing bar in a section or on the elevation the corresponding bar will be marked in the table. Several bars can be marked at the same time by **frame** (mark the first corner with the left mouse-button, keep it pressed down, drag and release it). Optional data can be changed and added in the table. The editing is made easier with the help from the buttons.

With the **Copy** button current bar-data can be copied and inserted in any optional position. When the copying has been activated the marker will change in order to mark that insertions are possible, and this can be repeated an optional number of times. The copy-mode will end by once again pressing the button, which is now marked **End copying**.

With the **Insert** button a copy of the marked bar will be inserted between this bar and the previous bar in the table. The bars should be written into the table in layers and counter-clockwise to maintain a good function of the insert function.

You can remove the marked bar from the table with the **Delete** button.

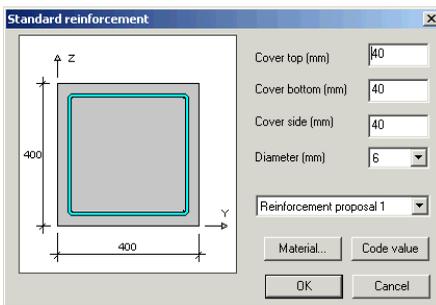
Marked bars can be split into two parts with the **Split** button. The position of the partition is given in the **X**-coordinate text box.

With the **Modify** button positions and lengths of marked bars may be altered. New positions and lengths are stated in the adjacent text boxes.

A bar can be extended to full length with the **Adjust Measurements** button.

2.1.2.2 Standard reinforcement

With the **Std reinforcement** button you open a window that will make it easy to fit the reinforcement to the current type of section.

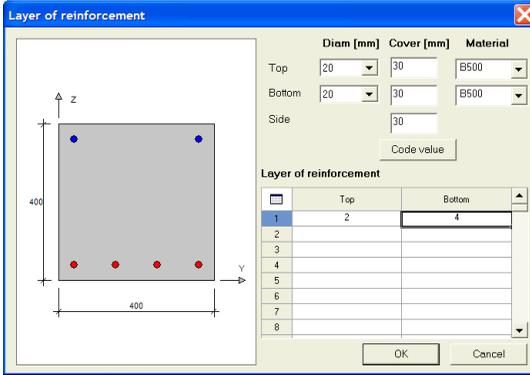


You can state reinforcement diameter and covers in this window, and with the **Material** button the window for **Material - Reinforcement** (see ch. 2.1.1.3 above) can be activated for selection of strength data for the reinforcement.

From the proposal list standard bar configurations can be selected.

With the **Code value** button the reinforcement covers can be restored to code-related values.

2.1.2.3 Layer of reinforcement

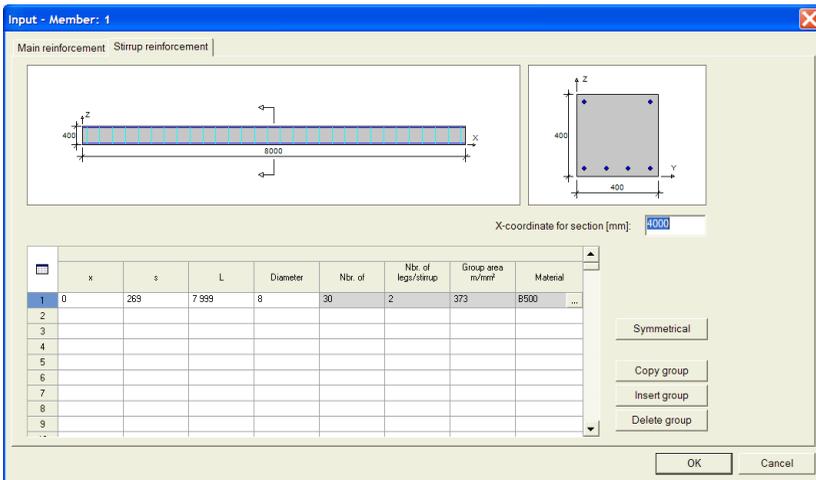


With the **Layer of reinforcement** button you open a window in which it is possible to input or adjust reinforcement layers.

You can state reinforcement diameter and covers in this window, and with the **Material** button the window for **Material - Reinforcement** (see chapter 2.1.1.3 above) can be activated for selection of strength data for the reinforcement.

activated for selection of strength data for the reinforcement.

2.1.2.4 Stirrup reinforcement



Under the **Stirrup reinforcement** tab stirrup reinforcement is displayed graphically as well as in tables. An optional section can be displayed by (continuously) dragging the section arrow with the mouse pointer, or by giving a numerical

X-coordinate value for the position of the section. By pointing at a stirrup group in a section or on the elevation the corresponding group will be marked in the table.

With the **Copy material** current material will be copied into all groups.

With the **Copy group** button current group-data can be copied and inserted at any optional position. When the copying has been activated the marker will change in order to mark that insertions are possible, and this can be repeated an optional number of times. The copy-mode will end by once again pressing the button, which is now marked **End copying**.

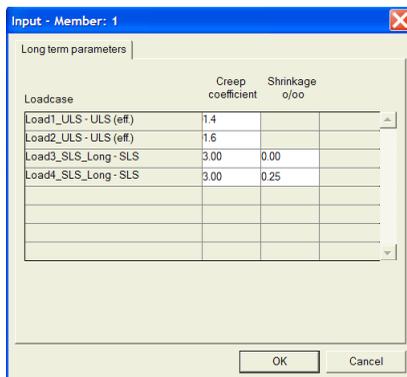
With the **Insert group** button a copy of the marked group will be inserted between this and the previous group in the table. The groups should be written into the table in order.

You can remove the marked group from the table with the **Delete group** button.

A group will be inserted symmetrical around the centre of the member with the **Symmetry** button.

2.1.3 Long term parameters

Load case dependent concrete data is stated in the **Long-term parameters** window.

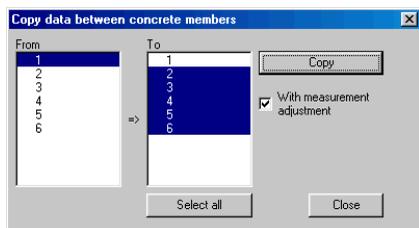


ULC load cases and SLS load cases defined as long-term loads (see **Frame Analysis** chapter 2.1.2.6) will automatically be copied into the table. This means that the load cases must be created before the parameters can be stated. The parameters for creep and shrinkage can now be stated individually for each load-case and frame-member.

In earlier versions of the program creep values could not be defined for ULS load cases. The way to consider creep was then to tie an ULS load case to a long term SLS load case by defining a dependency between these load cases and then an effective creep value was calculated with regard to the relationship between the ULS load and the SLS load as well as the relationship between the ULS E-module and the SLS E-module.

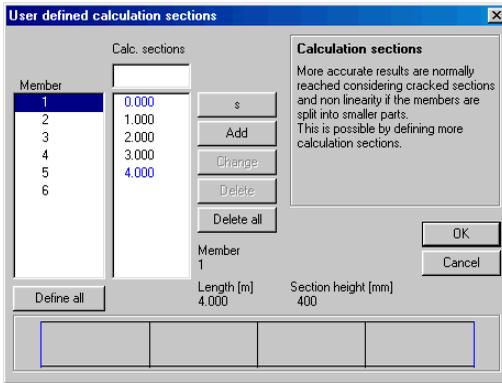
In version 6.0 the user can define creep values also for ULS load cases. For ULS load cases it is the effective creep number that should be defined. It is also possible to let the program calculate an effective creep value as in earlier versions of the program by defining a dependency with a long term SLS load case. In that case the original creep value should be defined also for the ULS load case and then an effective creep value will be calculated with regard to the relationship between the ULS load and the SLS load as well as the relationship between the ULS E-module and the SLS E-module. If the creep value for the ULS load case is zero no creep effects will influence the analysis for this load case although a dependency is defined. This is different from the way earlier versions of the program handled this effect.

2.1.4 Copy concrete data



All concrete data can be copied from one member to another to the, in the lists, mentioned extent. If *measurements adjustments* check box has been marked some adjustments of positions etc. will be performed during the copying. See measurements adjustments for main reinforcement and stirrup reinforcement.

2.1.5 Calculation adjustments



Consideration is taken to cracking and other non-linearities during the calculation by adjusting the effective cross section properties for each frame member in accordance with section controls performed at each member end. On the members the user can state a closer section control than the original, by choosing more sections to be calculated for each member.

With a closer section control chosen, the original members are split in more members internally, giving a more accurate calculation that naturally will take more time to execute. See also **Design** chapter 2.3.

Single section control positions can be stated. You can use *spacing* for a quicker input.

Note, that too short member length(s) after splitting may degrade the calculation result. It is recommended to not use shorter splitted member lengths than member height and/or approximately 1/10 of original member length.

Note also, that the program inserts automatically internal sections at concentrated loads and these sections cannot be removed. In stead remove other sections to close these.

2.2 Calculate

The screenshot shows the 'Calculate' dialog box with the following settings:

- Second order analysis
- All loadcases
- Buttons: Calculate, Cancel
- No. of displayed sections: 20
- Convergence cond. (%): 2.00
- Max. no. of iterations: 10
- Button: Standard
- Section: Calculation parameters for concrete
- Convergence cond. (%): 0.10
- Max. no. of iterations: 20
- Min. stiffness red. (%): 10
- Cracked section analysis
- Button: Standard

When **Calculate** has been selected to perform frame analysis with concrete members, the following input-box will be displayed on your screen.

In addition to load cases and so on, according to **Frame Analysis** chapter 2.2, convergence conditions, maximum number of iterations and lowest reduced stiffness (in percent of original) connected to the concrete calculation will be stated.

Calculation with considering cracking

Use check box to select method of calculation to be used in the frame analysis (not available at design of initial reinforcement).

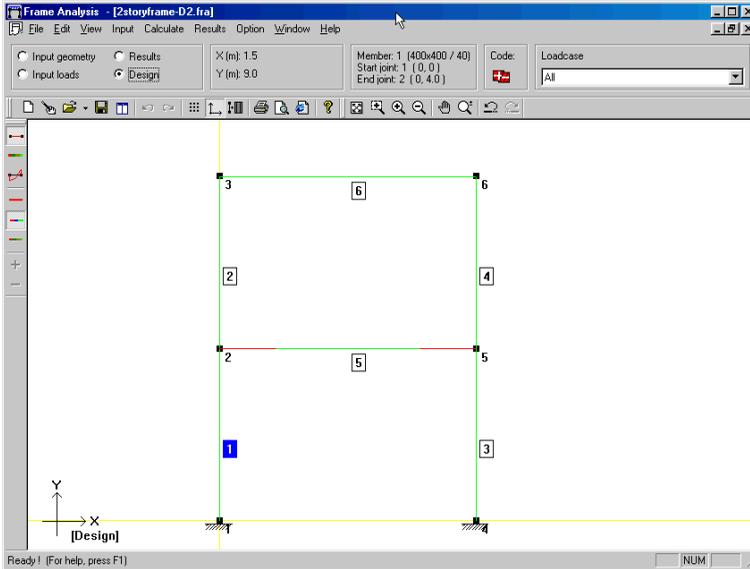
When the calculation considering cracking has been selected the calculation will be performed with consideration to changes of stiffness due to cracking. This often calls for an iterative calculation, see chapter 3.

The calculation will be performed with stiffness for uncracked sections in other cases. Consideration will however be taken to the reinforcement. The method of calculation can be used as a reference to the cracked result.

2.3 Design

When the calculation has been completed the **Design** program mode is enabled, and the appearance of the main menu changes as seen below. The **Design** option in the main menu makes it possible to define any additional information that may occur for a design control as well as study the results of the calculation.

Note! This mode is only available if **Concrete Module** has been installed.



- File** Contains information for different functions such as open saved information, save new information and plot. Several of the **File** options are identical to other Windows programs.

- Edit** Contains editing tools to undo and redo.

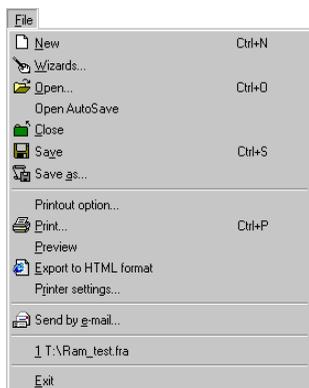
- View** This function is used for controlling the graphical drawing of the frame on the screen. It also makes it possible to display/hide different fields/tools, and to switch from one program mode to another.

- Input data** This menu option helps the user to define the problem in a numerical way. Note that the problems often can be defined graphically with some help from the graphical menu **Geometry tools**. The user can also mix numerical input data with graphical input data.

- Calculate** This option is used for dimensioning the chosen element.

| | |
|----------------|--|
| Results | This option is used for studying the, from the calculation, generated results. |
| Option | Contains functions to initiate the drawing area in a way that suits the current problem. |
| Window | Shows the current window. |
| Help | Contains the complete manual of the program. |

2.3.1 File



| | |
|----------------|---|
| New | Starts a new calculation. The short cut is Ctrl+N. |
| Wizards | The user can choose from several pre-defined frameworks and frames. See the Frame Analysis manual for a more thorough description. |
| Open | Retrieves input data from an earlier calculation. A list on saved input data files (files that end with fra) is shown. The short cut is Ctrl+O. |

| | |
|------------------------|---|
| Open Auto Save | Retrieves input data that has been saved automatically (in the TEMP-catalogue). This function is normally not needed. |
| Close | Closes current activity. |
| Save | Saves input data for the current calculation in the file with the current filename. Save is not used until after having used Save As . The short cut is Ctrl+S. |
| Save As | Saves input data for the current calculation in a file with a filename different from the current. |
| Printout Option | With this option you choose input data and the result that will be printed. |

| | |
|-------------------------|--|
| Print | Used for printing the current document. The short cut is Ctrl+P. |
| Preview | Makes it possible to, on screen, study the exact outcome of the chosen printout. |
| Printer Settings | This function is used for specifying the printout when it comes to type of printer, paper width and so on. |
| Send by e-mail | Used for sending current input data via e-mail. |
| Exit | Ends Frame Analysis . |

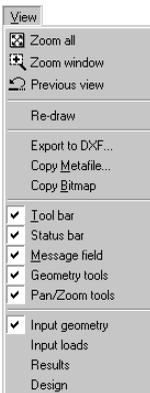
2.3.2 Edit



Undo You can undo a command or an action with this option, and reverse one step. This can be repeated several times. The short cut is Ctrl+Y.

Redo This option is used when reversing the effects of the **Undo** command. It can be repeated several times. The short cut is Ctrl+Z.

2.3.3 View



View Frame Presents the frame drawn in a maximum use of the window.

Previous Makes it possible to revert to the previous zoom position.

Zoom A specified area of the frame can be enlarged.

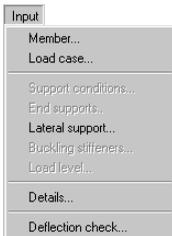
Re-draw Re-draws the frame.

Export to DXF With this option current geometry can be read by other programs, e.g. AutoCAD, which can handle DXF Format.

Copy Metafile The contents of the drawing area are copied to a metafile.

| | |
|----------------------------|---|
| Copy Bitmap | The contents of the drawing area are copied to the clipboard where it can be retrieved by other programs. |
| Toolbar | Makes it possible to hide and restore the tool palette. |
| Status bar | Makes it possible to hide and restore the status bar. |
| Message field | Makes it possible to hide and restore the message field. |
| Design tools | Makes it possible to hide and restore the design tools. |
| Input data geometry | This option is used for activating the geometry mode. |
| Input data loads | This option is used for activating the load mode. |
| Results | This option is used for activating the result mode. |
| Design | This option is used for activating the design mode. |

2.3.4 Input data

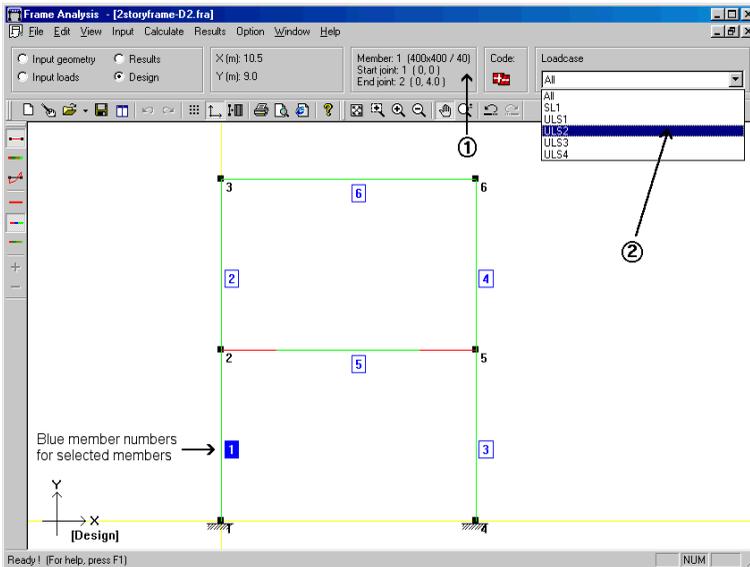


| | |
|---------------------------|--|
| Member | This option is used to select which members are to be designed. |
| Load cases | This option is used for selecting which load cases are to be designed. |
| Support conditions | Not active with concrete members in current version. |

| | |
|----------------------------|---|
| End supports | Not active with concrete members. |
| Lateral support | Not active with concrete members in current version. |
| Buckling stiffeners | Not active with concrete members. |
| Load level | Not active with concrete members. |
| Details | Displays a detailed picture on the current member. |
| Deflection check | Makes it possible to enable and select conditions for deflection control of active or selected members. |

2.3.5 Calculate

The **Calculate** menu option starts the design control calculation of currently selected members. Selected members have blue coloured member numbering. The members will be coloured according to information at **Degree of utilization with colours** see 2.4.12 below. This will help you to, with a quick look, see where measures have to be taken. By shifting to other calculated load cases you swiftly can control if the load-bearing capacity of the structure is sufficient for all load cases.



By the arrow (1) you can see the currently active member, i.e. the member for which results can be shown. You can swiftly change load cases with some help from the arrow (2).

2.3.6 Results



Material Shows material types and design values for concrete and reinforcement.

Section properties Shows a graphic representation of current profile with belonging geometrical section properties.

Capacity Displays the current member capacity with regards to moment and shear force.

Control Displays for Ultimate Limit State load cases all code controls for load-bearing capacity of the active member. A deflection control might be displayed for Serviceability load case if it has been defined for the current load case.

Degree of utilization with colours This is used for getting a general view over the degree of utilization of members according to user given limits.

Degree of utilization table Shows (for all members) the value of the control that utilizes the member the most.

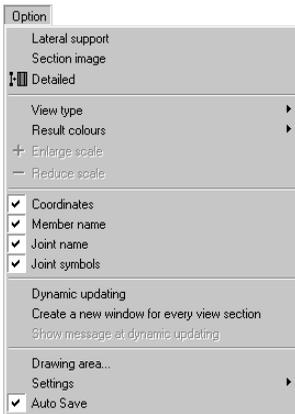
Table - bending... Shows bending results tabulated. In all sections section forces, moment capacity, strains and so on are displayed.

Table - shearing... Shows shear results tabulated. In all sections section forces, shear force capacity, shear reinforcement and so on are displayed.

Table - crack data... Shows crack results tabulated. In all sections crack state, cross section data, crack moment, crack width and so on.

Interactive graph... Graphically displays the correlation between axial force and moment capacity with regards to different bending axes.

2.3.7 Options



Lateral support

Not active with concrete members in this version.

Section image

Displays/hides the section image of the member.

Detailed

Displays/hides the members drawn according to scale with regards to chosen cross section including reinforcement.

Type of presentation

Normal displays the members of the frame in thin coloured lines to show degree of utilization.

Graphs display the degree of utilization as graphs along the frame.

Thicker members display the members of the frame in thick coloured lines to show degree of utilization.

Result colours

Monochromatic displays the degree of utilization in graphs.

Three colours display degree of utilization.

Red-green tint displays degree of utilization.

Enlarge scale

This function is used for enlarging the scale of a graph result.

Reduce scale

This function is used for reducing the scale of a graph result.

Coordinates

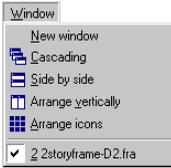
This function displays/hides the current coordinates of the cursor.

Member names

Displays/hides the member names.

| | |
|---|---|
| Joint names | Displays/hides joint names. |
| Joint symbols | Displays/hides joint symbols. |
| Dynamic updating | This function updates the calculation automatically when a change of input data has been made. This option builds on the assumption that every view has been assigned a window of its own. |
| Create a new window for every view | Here view means Input data Geometry , Input data Loads , Results and Design , which by this option are assigned windows of their own. |
| Drawing area | This function is used for setting drawing-area size, distance between points (grids), origin of coordinate axes, resolution of the grid and snap distance to the joint/member (pixels). |
| Settings | <p>Start settings: This function is used for specifying the document- and language settings.</p> <p>Paths: This function is used for defining paths to data files such as section database, codes or templates.</p> <p>Window title: This function is used for specifying the text in the window title-line. (Note that title or filename must be included.)</p> <p>User/Signature: This function is used for removing users and to show how the automatic generation of the signatures is to be performed.</p> |
| Auto Save | This function is used for creating a document restore file automatically after a 10-minute time interval. The restored file may contain information that has not been saved and would have disappeared from the original document. The restored file does not replace the Save command. The document must still be saved when you have finished working with it. |

2.3.8 Window



New Window This function opens a new window with the same example thus giving opportunity for the user to have several pictures, belonging to the same example, visible at the same time.

Cascading

This function is used for arranging the current document windows in a way that makes them cascade each other so that the title-lines are visible.

Arrange Horizontally

This function is used for arranging the current document windows so that they all can be shown in full.

Side-by-Side

This function places all open document windows side by side, so that all of them can be seen at the same time.

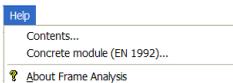
Arrange Icons

This function is used for arranging the different calculation-example icons that might be on the screen.

√ 2 2storyframe-D2.fra*

The marking shows that the window that, e.g. represents the **2storyframe-D2** is active.

2.3.9 Help



Contents The **Frame Analysis** manual can be accessed directly in the program by this option.

Active Concrete Code

The **Concrete Module** manual can be accessed by this option.

Active Timber Code

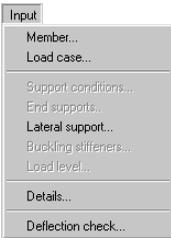
The **Timber Module** manual can be accessed by this option.

Active Steel Code

The **Steel Module** manual can be accessed by this option.

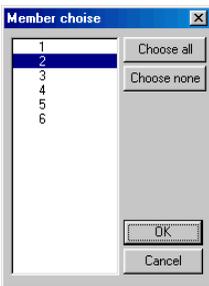
About Frame Analysis This function displays information, such as version number, about the program.

2.4 Input data



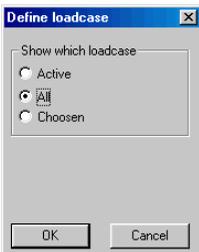
The Input data menu option makes it possible to define possible additional information for a design. Under **Input data** the **Members**, **Load cases**, **Lateral support**, **Details** and **Deflection check** options are active during concrete design.

2.4.1 Members



The **Members** option displays a dialog-box in which you can select requested members.

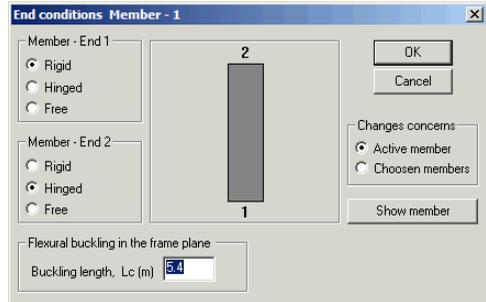
2.4.2 Load case



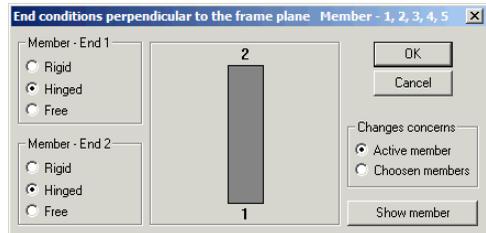
In **Load case** you choose what load cases is to be calculated for design check.

2.4.3 Support conditions

In the dialog for **Support conditions** buckling length is given regarding buckling in frame plane if frame analysis has been performed according to 1st order theory. Here is also defined support conditions regarding buckling out of plane. Initially all members defined with the tool **Member** in **Frame Analysis** are supposed to have forked support conditions in both ends. This means that the buckling length is equal to the member length.



Note! A joint that has been defined with the tool **Member** will automatically be regarded as stabilized with regard to buckling out of plane. If no stabilizing sideways is wanted use instead the tool **Unsupported joint**.



If design is performed according to 2nd order theory section forces only support conditions regarding buckling out of plane shall be defined.

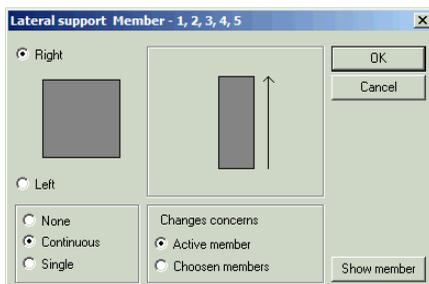
At changing only the active member or selected members are affected depending on the before said. In the above picture only **1** member is selected which is seen in the title row of the dialog. In order to be possible to give support conditions at least one member must be selected. If it shall be possible to set a free end the other end must be fixed.

Buckling length will also be used to calculate initial bow imperfection out of plane.

2.4.4 Lateral supports

In the dialog for **Lateral supports** the user can define the lateral support or bracing between joints to reduce instability out of frame plane.

Lateral supports is defined as continuous or point support. When a lateral support has been defined it will prevent the member to bend sideways in the support point. This requires the side supporting structure to have enough strength to be able to prevent the lateral movement.

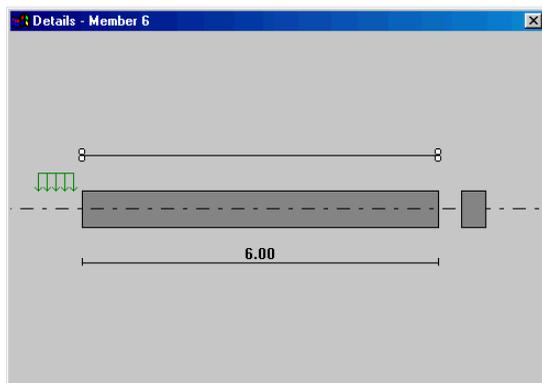


In the **Concrete Module** lateral instability is not considered but only buckling. Thus it is will give the same result to stabilize top as to stabilize bottom of section or both.

Max number of lateral point supports allowed are **10**. The point supports may not be put closer than **L/10**.

Lateral support of circular section can not be defined.

2.4.5 Details



A window opens, in which current member/members are shown as a beam or a column, when the **Details** option has been activated.

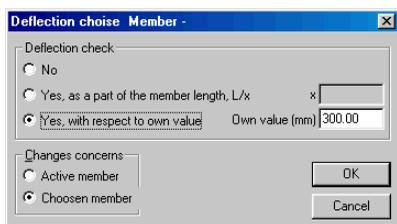
Details can only be defined if the stiff direction of the member is in the frame plane. Current end conditions are shown in

the picture (examples, see beside). The defined details are only valid for the selected members and at least one member must be selected in order for details to be stated.

The picture above displays a column member whose left side has been stated as continuous lateral braced. At continuous lateral bracing it is assumed that the bracings are being distributed over the member's entire length thus not allowing for lateral buckling.

The picture above displays how upper edge point bracings for a beam member is shown. Buckling between bracings can occur when using point bracings. The buckling length will be calculated with regards to all defined bracings when calculating the load-bearing capacity due to bending instability in the weak direction.

2.4.6 Deflection check



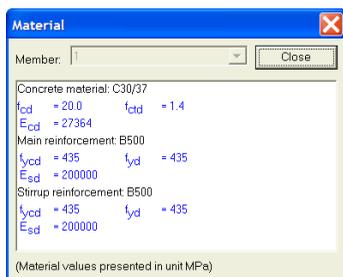
You state whether you want to have a deflection check for active or selected members in the **Deflection check** dialog box. The deflection check can then be performed with respect to a factor of current span length or a stated value. In the latter case you will get the same criterion for all selected members.

2.4.7 Results



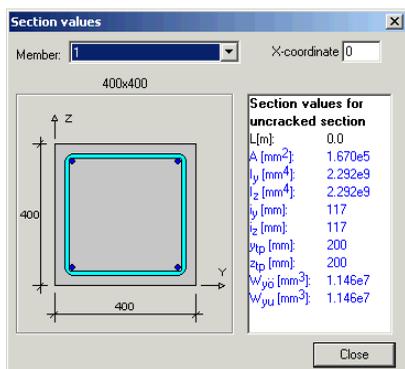
The **Results** menu option makes it possible to study the results generated by the calculation. Under **Results** you can find the following options: **Material**, **Section properties**, **Capacity**, **Control**, **Degree of utilization with colours**, **Degree of utilization table**, **Table-bending**, **Table-shear**, **Table-crack data** and **Interaction graphs**.

2.4.8 Material



Material design strengths are shown in the **Material** dialog box.

2.4.9 Section properties



The **Section properties** option displays a graphic picture of the current profile with belonging geometrical properties in uncracked state including reinforcement.

2.4.10 Capacity

The **Capacity** option displays a dialog box where the capacity of the active member, with regards to current section forces, is shown. The shown result also depends on possible lateral bracing.

2.4.11 Code Check

The **Code Check** option displays (for the active member at Ultimate Limit State) a picture in which all controls of the load-bearing capacity according to **Active Concrete Code** are shown as well as a deflection control for Serviceabil-

ity Limit State if such has been defined for the current load case. Here it is also shown where the most critical section is located, and if upper or lower edge is critical. For vertical members the left side corresponds to the upper edge. Checks that might indicate that the load-bearing capacity is insufficient are shown with red text.

2.4.12 Degree of utilization with colours

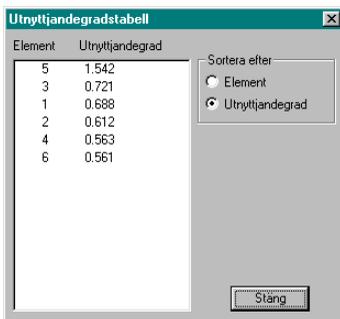
When the design has been completed the included members will get different colours depending on how the result turns out. The members that pass all controls will get a green colour and the others will get a red colour as default.



You can also get members coloured in blue according to the **Degree of utilization with colours** option. The purpose with this can e.g. be to swiftly survey, not only which members will pass but also to see which members are being utilized over a user-given limit.

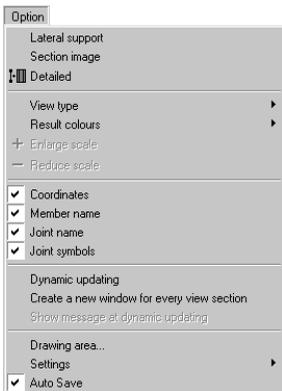
In the picture above an example is shown where all members that are being utilized less than 50% are being green coloured, those that are being utilized 50-100% are being blue coloured, and those that don't have enough capacity are being red coloured.

2.4.13 Degree of utilization table



In order to quickly get an opinion on in what grade the different parts of the structure are being utilized, the **Degree of utilization table** option can be used. At this a table will be displayed, showing the value (for all members) of the control that utilizes the member the most. The table can be sorted according to utilization or order of member number.

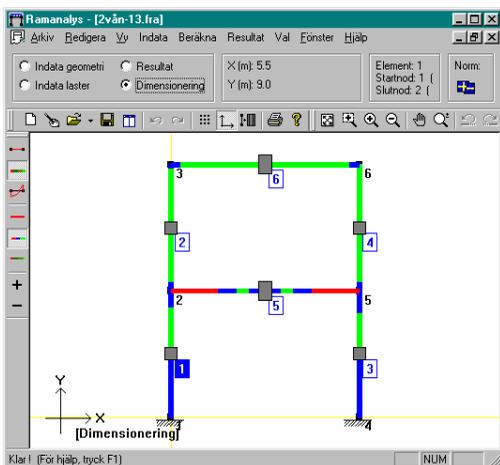
2.5 Option



2.5.1 Lateral support

With this option support against flexural buckling and lateral torsional buckling out of the frame plane can be defined.

2.5.2 Section image



If the **Section image** in the **Option** menu is active the current cross section will be shown on the image as seen below:

3 Methods of calculation

3.1 Design methods with regard to buckling

The design is being performed according to [1] along with all code prescribed controls. Flexural buckling in the frame plane can be performed according to 1th and 2nd order theory. For buckling out of the plane frame 1th order theory is used. The user chooses desired method in **Frame Analysis**.

3.1.1 The First Order Theory

The following things are valid when designing according to the first order theory:

During a section force calculation consideration will be taken to the influence of cracking according to chapter 3.2. However, the influence of the frame deflection on force and moment distribution will not be considered, which means that instability will not be considered in the frame analysis.

The capacity with regard to flexural buckling is calculated according to [1] 5.8.7 *Method based on nominal stiffness* or [1] 5.8.8 *Method based on nominal curvature* depending on the user choice. In the frame plane the buckling length is defined by the user and out of the plane the buckling length is calculated from support conditions defined by the user. The effect of initial bow imperfection is already considered and has not to be defined separately.

3.1.2 The Second Order Theory

The second order theory refers to that theory considers the influence of deflection on force and moment distribution within the frame. There is no need to think of buckling lengths in the frame plane when using this method since this will be regarded during the frame calculation with a second order analysis of the structure as well as the member. The slenderness effects will therefore form a part of the second order theory section forces and if the frame or its members are too slender the frame will be unstable and the calculation is not possible to fulfil.

The limit for load-bearing capacity, with regards to slenderness (bending instability within the frame plane), is then shown in the section control that is performed for the second order section forces according to [1] 5.8.6 **General method**. When all members pass the section control there will be enough buckling safety within the frame plane for each member as well as for the frame as a whole.

Calculations according to the second order theory must have been selected in **Frame Analysis** in order to perform the design in this way in **Concrete Module**. Further when using this method the following demands must be fulfilled:

The stiffness of the frame must be reduced with consideration to cracking, see chapter 3.2. It is also possible to state value of lowest reduced stiffness to be used in **Frame Analysis**, which may have influence on the calculation result.

Consideration shall be taken to initial bow imperfection. Use the tool in **Frame Analysis** for this to state which members shall have initial curvature due to imperfection and let it act at least on compression members. It is particularly important for hinged members when they are loaded only with axial force or not at all.

When working with sway-frames (or isolated columns) consideration must also be taken to initial inclination from the vertical of the frame. **Frame Analysis** will not automatically consider this, so the user must choose another method in order to include effect of initial inclination. A suitable method is to add fictive horizontal loads corresponding to the effect of the inclination.

3.2 Calculation of concrete frames with regard to cracking

Unlike other ordinary construction materials, e.g. steel, the characteristics of concrete make calculations of in particular statically undetermined structures complicated. Principally it is cracking and the following changes in stiffness that makes things more difficult. The non-linearity at higher strains and long-time effects such as creep adds to the complexity.

3.2.1 Calculation principles

The calculation of the frame is performed according to theory of elasticity as described in **Frame Analysis** 3.1.

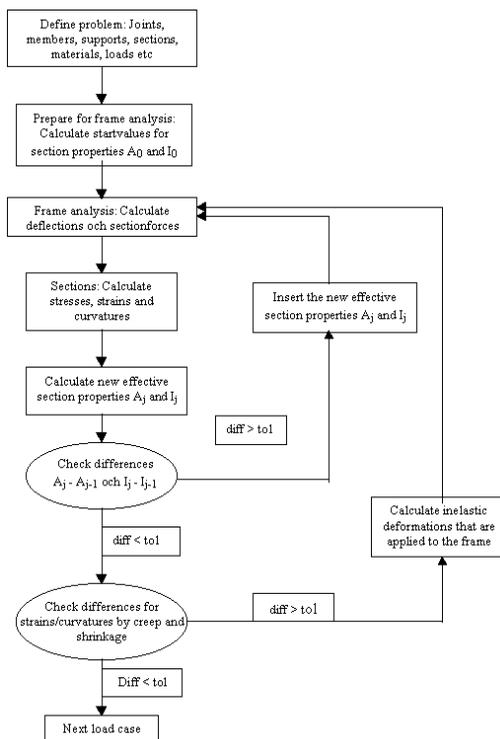
During the calculation member stiffnesses, that have been calculated from strain and curvature obtained from a thorough (possible non-linear) section-analysis with section forces from the frame-calculation, will be used. The method can be seen as an iterative method of secant stiffness.

Other non-linear behaviour at cracking is condensed to strains and curvature, which form a part of the iteration as load between the nodes (corresponding to thermal load).

Non-linear strains of creep and shrinkage are converted to loads between the nodes (corresponding to thermal load). Since the size of creep-strains and curvature depend on the distribution of section forces the calculation of creeping also calls for iteration.

Remaining deflections from earlier load cases can also be added.

Flowchart:



3.2.2 Effective stiffness

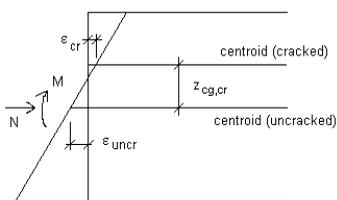
A new effective stiffness, in the form of A_{eff} and I_{eff} from the strain and curvatures given from section calculations performed with current section forces, will be calculated for each member after every iteration-step in the calculation of the frame. Section properties are being calculated for the part of the strains and curvatures that remain when creep, shrinkage, temperature and so on have been deducted. We will then get:

$$A_{\text{eff}} = N / E_c / \varepsilon$$

$$I_{\text{eff}} = M / E_c / \kappa$$

- where:
- A_{eff} = effective area,
 - I_{eff} = effective moment of inertia,
 - E_c = modulus of elasticity for concrete,
 - ε = strain in the member's centroidal curve,
 - κ = curvature of the member.

If the section calculation is non-linear due to the stress-strain curve of the materials, mainly in Ultimate Limit State, there will be an effective stiffness that, adapted to the increasing strain and curvature, is smaller than it is at linear elastic conditions.



Since A_{eff} and I_{eff} above are of physical significance as section properties, they must not be allocated negative values. This means that measures must be taken when N or M equals zero in order to avoid dividing zero with zero. The original values for A and I will be used in this situation.

The centroid of the section will be moved from its original location in the un-cracked section during cracking. The strain must be calculated in the cracked state in order to get a strain value that corresponds to A_{eff} . This means that a cracked section gives:

$$\varepsilon = \varepsilon_{\text{cg,un-cr}} + \kappa z_{\text{cg,cr}}$$

Note that the curvature does not change.

3.2.3 Inelastic strain at cracking

In the calculation of the frame the members are represented by their centroidal curve, which coincides with uncracked centre-of-gravity line. The centroids will be moved to a new position when a member cracks, which means that the centre-of-gravity line of the member will differ from the original centre-of-gravity line for an uncracked member. This must be considered when the strain-curvature results from the section calculation are translated into effective stiffness.

As described above (see chapter 3.2.2) the conditions displayed in the figure (axial compressive force but tension in the centroidal curve of the member) cause trouble when calculating the effective area. This is more compatible to the strain in the centre of gravity of a cracked section. However, the lengthening of the centroidal curve will not be compatible to axial compressing strain, i.e. compatibility in strain is not at hand. In order to restore the compatibility, the program chooses to interpret the difference in strain between the frame calculation and the section analysis as an inelastic deflection that is added to the frame.

The difference in strain for a member is obtained from:

$$\Delta\varepsilon = \varepsilon_{\text{cg,uncr}} - \Delta / L$$

where: $\varepsilon_{\text{cg,uncr}}$ = an average strain in the centroidal curve of the member, exc. creeping, shrinkage and thermal elongation,
 Δ = an average strain in the centroidal curve of the member, exc. creeping, shrinkage and thermal elongation,
 L = member length.

During iteration $\Delta\varepsilon$ is added to $\Sigma\Delta\varepsilon$ from the previous step. When a displacement is $\Delta\varepsilon = 0$ during a new iteration step, the strains within the frame calculation and section analysis are corresponding.

3.2.4 Inelastic strain and curvature of creep and shrinkage

If creep or shrinkage has been defined a calculation of the effects on equilibrium of forces and the strains will be performed during the section analysis. The parts of strain and curvature that correspond to creep and shrinkage are separated and

then added to the frame as an inelastic deflection. When the frame is statically undetermined, the added inelastic deflections will generate section forces that will be added to section forces from other load-effects. It is then obvious that the effect of creep and shrinkage must be calculated with iteration. See flowchart.

3.2.5 Inelastic strain and curvature from previous load cases

Creep and shrinkage result in that deflections remain in the frame after unloading. These deflections are stored for all concerned load cases during the calculation, and can be re-activated with **Dependence** (see **Frame Analysis** 2.1.2.6), i.e. the calculation result of a load case can be made dependent on a previous calculated load case. This way the effect of a long-time load in one load case can be made to work in another. When calculating the new load case the inherited deflections will be considered as added deflections in the same way as any other added deflections.

3.2.6 Cracking in previous load cases

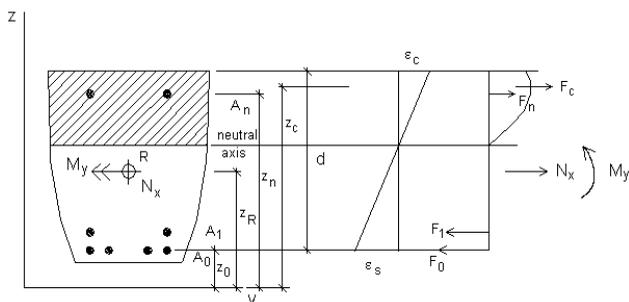
Possible cracking in a load case is stored during a calculation. Through **Dependence** (see **Frame Analysis** 2.1.2.6) the cracking from a previous load case can be inherited, which might affect the analysis of the current load case.

3.3 Design

3.3.1 Bending reinforcement

Design of required (tensile and/or compression) reinforcement in a section providing enough flexural capacity for specific section forces is one of the keystones in ultimate bending design. This can be done in several ways, e.g. as a trial and error method, as a coefficient method or as a direct method. The suitable method depends on section type, moment vector direction, reinforcement rules etc. Here we assume bending axis to be parallel to **y**-axis or **z**-axis and use a direct method to get the most efficient result. At biaxial bending the design

problem has generally more than one solution and we then use an approximate method, see ch. 3.3.1.6.



Before the design calculation can start conditions for the design must be set. These include concrete and reinforcement quality, bar sizes, cover and spacing rules as well as maximum and minimum rules for bar sizes and number of bars.

From cover and spacing rules we get possible positions of reinforcement layers both in bottom and top of section as well as possible numbers of bars in each layer at a given bar size.

From material properties we get maximum compressive strain in concrete and maximum tensile strain and yield strain in reinforcement. As we have assumed bending axis to be parallel to y -axis or z -axis it follows that the neutral axis is also parallel to y -axis or z -axis. It is now possible to establish various strain distributions for the section within the limits given by maximum strain values. To each combination of strain distribution and bar positions will correspond a set of internal normal- and moment forces. An iteration procedure is then mostly necessary to establish a suitable set of bars, which together with an appropriate strain distribution will give the requested capacity.

3.3.1.1 Balanced reinforcement and balanced moment

To start with a strain distribution corresponding to $\epsilon_c = \epsilon_{cmax}$ and $\epsilon_s = f_y / E_{sk}$ is chosen. When calculating internal forces in section the method proposed in ch. 3.4.3.1 is used. From start no reinforcement area has been set, so the strain distribution will only give the compression force F_c in the concrete and its moment M_c with reference to the point R , which is the point of action for the exter-

nal section forces. Equilibrium in a direction perpendicular to the section plane will then give the required tensile force in reinforcement and thus the corresponding reinforcement area. We get (forces are positive at tension):

$$F_s = N_x - F_c$$

and the balanced reinforcement area:

$$A_{s0bal} = F_s / \sigma_{s0}$$

where $\sigma_{s0} = f_s(\epsilon_s)$ is calculated using the stress-strain relation for steel (see ch. 3.4.3.3).

If $F_s < 0$ we set $F_s = 0$ because a negative value does not conform to the chosen strain distribution.

The moment capacity with balanced reinforcement will be the balanced moment:

$$M_{bal} = M_{cap} = M_c + F_s (z_R - z_0)$$

Thus far we have a moment capacity, which corresponds to a reinforcement area in one lower layer. If current section force M_y is not equal to M_{bal} various steps of action have to be taken depending on whether $M_y < M_{bal}$ or not.

In later iteration steps additional reinforcement layers (normally filled up) may be present in tensile side (and compression side) and thus contribute to the internal forces. These layers will be regarded as part of the section and **balanced** reinforcement in the lowest, not filled layer can be calculated gradually. Equilibrium equation is modified to:

$$F_s = N_x - F_c - F_t$$

and the balanced moment in this case:

$$M_{bal} = M_{cap} = M_c + M_t + F_s (z_R - z_0)$$

3.3.1.2 Reinforcement with compression bars

When $M_y > M_{bal}$ in most cases we get the most economic reinforcement (= smallest total area) with reinforcement in compression zone and increased tensile reinforcement.

The additional force in upper layer **n** and lower layer **0** will then be:

$$\Delta F = (M_y - M_{bal}) / (z_n - z_0)$$

Calculate strain in upper layer **n** and corresponding stress σ_{sn} using stress-strain relation for steel as above:

$$\sigma_{sn} = f_s(\epsilon_n)$$

Calculate the concrete stress σ_{cn} at the position of layer **n** using the stress-strain relation for concrete (see ch. 3.4.3.2).

Reinforcement area in compression zone will be:

$$A_{sn} = \Delta F / (\sigma_{sn} - \sigma_{cn})$$

and tensile reinforcement area will be:

$$A_{s0} = A_{s0bal} + \Delta F / \sigma_{s0}$$

As the reinforcement area corresponds to an integer number of bars, we will not get the most economic reinforcement if we put in bars both in compression zone and tensile zone corresponding to respective zone area. Instead we choose to round the compression zone area to an integer number of bars and calculate the tensile area corresponding to this rounded area. We get:

$$A_{sadjusted} = \text{round}(A_{sn} / A_{bar}) A_{bar}$$

where **round** means round to higher or lower value in usual way.

$$A_{s0adjusted} = A_{s0} - (A_{sn} - A_{sadjusted}) (\sigma_{sn} - \sigma_{cn}) / \sigma_{s0}$$

3.3.1.3 Underbalanced reinforcement

When $M_y < M_{bal}$ the balanced reinforcement area in tensile zone is too large and must be decreased to give better reinforcement economy.

As the balanced reinforcement area is calculated with $\epsilon_s \geq \epsilon_{yp}$ a decrease of area means also a decrease of tensile force in reinforcement. The decrease of force in compression zone and tensile reinforcement will then be:

$$\Delta F = (M_y - M_{bal}) / (z_c - z_0)$$

Required compression force will be given by $F_c = F_{cbal} + \Delta F$ (note that ΔF is negative in this case). With no reinforcement in compression zone, the area of the compression zone then must be decreased to maintain equilibrium, which will influence the position of F_c and thus z_c will be changed too. Consequently, an iterative procedure is required to calculate ΔF .

Iteration procedure:

The iteration is performed by estimating a new value of the compression zone depth. A suitable estimation at the i :th step is:

$$d_c(i+1) = d_c(i) M_y / M_{cap}(i)$$

Keeping the concrete strain constant will give a new value of the tensile strain ϵ_s by calculating:

$$\epsilon_s = \epsilon_c (d - d_c) / d_c$$

and we can calculate new forces F_c and F_{s0} (which include ΔF).

When a constant ϵ_c gives $\epsilon_s > \epsilon_{smax}$ instead ϵ_c must be decreased keeping $\epsilon_s = \epsilon_{smax}$ constant.

The iteration continues until $M_{cap}(i) = M_y$.

When ϵ_{s0} and F_{s0} have been calculated corresponding stress σ_{s0} is calculated using stress-strain relation and required reinforcement area will be:

$$A_{s0} = F_{s0} / \sigma_{s0}$$

3.3.1.4 Reinforcement at tensed section

At external tensile normal force ($N_x > 0$) the eccentricity of the normal force must be checked to be able to decide if tensile reinforcement is needed both in bottom and top of section.

The eccentricity of the normal force is:

$$e_N = M_y / N_x$$

When $e_N > z_R - z_0$ the calculation may be performed as in the usual way as described before (see ch. 3.3.1.2 - 3.3.1.3).

When $e_N < z_R - z_0$ equilibrium is not possible without tensile reinforcement in top as well as in bottom.

In the last case forces in top and bottom can be calculated directly. Forces in upper layer **n** and lower layer **0** is calculated as:

$$F_{s0} = [M_y + N_x (z_R - z_0)] / (z_n - z_0)$$

$$F_{sn} = [-M_y + N_x (z_n - z_R)] / (z_n - z_0)$$

We then set the strain distribution equal to ϵ_y both in upper and lower layer and use the stress-strain relation to calculate stresses in the layers.

In lower layer the tensile reinforcement area will be:

$$A_{s0} = F_{s0} / \sigma_{s0}$$

And in upper layer:

$$A_{sn} = F_{sn} / \sigma_{sn}$$

3.3.1.5 Without tensile reinforcement

When $M_y < M_{bal}$ the balanced reinforcement area is too large and in some cases not needed at all to establish equilibrium between internal and external forces.

The eccentricity of the normal force is:

$$e_N = M_y / N_x$$

If the balanced reinforcement area is ignored we can verify that if $e_N < M_{cbal} / F_c$ and $M_y < M_{cbal}$ no tensile reinforcement is needed. As $N_x < F_c$ it is always possible to find a strain distribution that satisfies equilibrium.

When $e_N > M_{cbal} / F_c$ or $M_y > M_{cbal}$ we start with assuming that underbalanced reinforcement is a correct solution. If we then at iteration find that for a suitable ϵ_s equilibrium requires that $\epsilon_s < 0$ no tensile reinforcement is needed and the iteration is changed to find a compression zone depth that satisfies equilibrium.

Iteration procedure:

The iteration is performed by estimating a new value of the compression zone depth. A suitable estimation at the i :th step is:

$$d_c(i+1) = d_c(i) N_x / F_c(i)$$

Keeping ϵ_c constant will give a new value of the tensile strain ϵ_s (although no reinforcement is assumed at the lower layer) by calculating:

$$\epsilon_s = \epsilon_c (d - d_c) / d_c$$

and we can calculate a new value of the force F_c .

The iteration continues until $F_c(i) = N_x$

When $M_y < M_c$ the capacity is sufficient without reinforcement for the obtained strain distribution.

When $M_y > M_c$ compression reinforcement (and maybe tensile reinforcement) is needed in order to get sufficient moment capacity.

The added force in upper layer n required to get $M_y = M_{cap}$ will then be:

$$\Delta F = (M_y - M_c) / (z_n - z_R)$$

Calculate strain in upper layer n and corresponding stress σ_{sn} using the stress-strain relation as above:

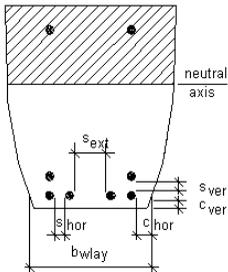
$$\sigma_{sn} = f_s(\epsilon_n)$$

Calculate also the concrete stress σ_{cn} at the position of layer n using the concrete stress-strain relation (see ch. 3.4.3.2).

Compression reinforcement area will be:

$$A_{sn} = \Delta F / (\sigma_{sn} - \sigma_{cn})$$

3.3.1.6 Arranging reinforcement bars



When calculating required reinforcement area for layers in bottom and top it must be checked that it is possible to arrange the corresponding number of bars in the layer considering rules for cover and spacing. If it is possible the design of reinforcement is finished. If required number of bars is larger than the maximum possible number of bar in a layer we have to settle with this number and increase the number of layers to give space for the missing bars. As the new layer is not so favourably situated as the one used in calculation the

calculation must be repeated to get a more correct value for the added layer.

When repeating the calculation the layers that are filled up will be treated as fixed and the added layers are viewed as bottom and top layer.

The calculation is repeated as long as the reinforcement must be moved to new layers. When there is no space for a new layer on the proper side of the neutral axis the possibility to put in more reinforcement is exhausted and this sets a limit to the capacity of the section.

3.3.2 Shear reinforcement

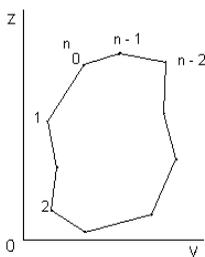
Design of shear reinforcement is performed by first calculating shear capacity as described in ch. 3.4.6.

The shear reinforcement area A_{sv} is converted to vertical 2-cut stirrups using selected diameter and appropriate spacing and shape depending on current section type.

3.4 Control

3.4.1 Section properties

3.4.1.1 Area properties of concrete gross section



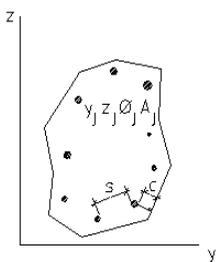
A general integral over an area of arbitrary shape is given as:

$$H_{mn} = \int y^m z^n dA$$

with m and n as positive integers. Setting m and n equal to 0 , 1 or 2 , the integral will give values of the area integrals A , S_y , S_z , I_y , I_z and I_{yz} which are defined as area properties.

At calculation all section types (open as well as closed) are converted to polygon shape and after that the section properties are easily calculated using numeric integration (see [6]).

3.4.1.2 Area properties of reinforcement in cross-section



Section properties for reinforcement composed of N bars with cross-section area A_{sj} and positions (y_j, z_j) are calculated as a summation:

$$H_{mn} = \sum_{j=1}^N y_j^m z_j^n A_{sj}$$

With m and n equal to 0 , 1 or 2 , the summation will give values of the area integrals A , S_y , S_z , I_y , I_z and I_{yz} which are defined as area properties.

3.4.1.3 Area properties of composite cross-section

Section properties for the composite section are calculated using the section properties of net concrete section (i.e. with area corresponding to reinforcement

subtracted) and section properties of reinforcement considering the difference in modulus of elasticity. This section is also referred to as the transformed section.

The difference in modulus of elasticity is expressed by the factor:

$$\alpha = E_s / E_c$$

Summation of areas and static moments around coordinate axes give property values of the transformed section:

$$A = A_{cn} + \alpha A_s$$

$$S_y = S_{cny} + \alpha S_{sy}$$

$$S_z = S_{cnz} + \alpha S_{sz}$$

Coordinates for the centroid of the transformed area are then easily found by:

$$y_{cg} = S_y / A$$

$$z_{cg} = S_z / A$$

Moments of inertia are then calculated using Steiner's theorems:

$$I_{ycg} = I_{cny} + \alpha I_{sy} - z_{cg}^2 A$$

$$I_{zcg} = I_{cnz} + \alpha I_{sz} - y_{cg}^2 A$$

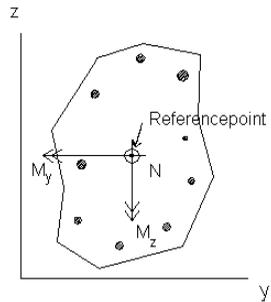
$$I_{yzcg} = I_{cnyz} + \alpha I_{syz} - y_{cg} z_{cg} A$$

3.4.2 Stresses and strains at linear section analysis

Section forces are assumed to act in the reference point **R**, which coincide with the centre-of gravity of the uncracked section.

At linear section analysis it is assumed that:

- a) Plane cross-sections remain plane at bending (Bernoulli's hypothesis).
- b) Only normal stresses occur in pure bending (Navier's hypothesis).
- c) A linear stress-strain relation applies for the normal stresses in every fiber (Hooke's law).



Formulated as equations (see also [5] and [6]) we get from a):

$$\epsilon = \epsilon_0 + \psi_y (z - z_R) + \psi_z (y - y_R)$$

where: ϵ_0 is the strain at the reference point \mathbf{R} ,
 $\psi_y = \partial\epsilon/\partial z$, and $\psi_z = \partial\epsilon/\partial y$, represent the curvatures in the \mathbf{xz} and \mathbf{xy} planes.

From conditions b) and c) we get:

$$\sigma = E \epsilon$$

$$N = \int \sigma \, dA$$

$$M_y = - \int \sigma z \, dA$$

$$M_z = \int \sigma y \, dA$$

Substitution of the expression for ϵ gives:

$$N = E (A\epsilon_0 + S_y\psi_y + S_z\psi_z)$$

$$M_y = - E (S_y\epsilon_0 + I_y\psi_y + I_{yz}\psi_z)$$

$$M_z = E (S_z\epsilon_0 + I_{yz}\psi_y + I_z\psi_z)$$

where \mathbf{S} and \mathbf{I} are static moments and moments of inertia for the section area around axis parallel to y and z -axes through reference point \mathbf{R} .

In matrix form it can be written:

$$\begin{Bmatrix} N \\ M_y \\ M_z \end{Bmatrix} = E \begin{bmatrix} A & S_y & S_z \\ -S_y & -I_y & -I_{yz} \\ S_z & I_{yz} & I_z \end{bmatrix} \begin{Bmatrix} \epsilon_0 \\ \psi_y \\ \psi_z \end{Bmatrix}$$

This equation can be used to determine the stress resultants $\{N, M_y, M_z\}^t$ when the strain or stress distributions are known.

The inverse can be used to determine for given values of $\{N, M_y, M_z\}^t$ the axial strain and the curvatures that define the strain distribution:

$$\begin{Bmatrix} \epsilon_0 \\ \psi_y \\ \psi_z \end{Bmatrix} = (1/E) \begin{bmatrix} A & S_y & S_z \\ -S_y & -I_y & -I_{yz} \\ S_z & I_{yz} & I_z \end{bmatrix}^{-1} \begin{Bmatrix} N \\ M_y \\ M_z \end{Bmatrix}$$

Multiplication of the strain distribution by E gives the stress distribution with the three stress parameters:

$$\begin{aligned} \sigma &= E \epsilon_0 \\ \gamma_y &= E \psi_y \\ \gamma_z &= E \psi_z \end{aligned}$$

and the stress in an arbitrary point as:

$$\sigma = \sigma_0 + \gamma_y(z - z_R) + \gamma_z(y - y_R)$$

3.4.2.1 Stresses in reinforcement

When the strain distribution over the section is determined it is it easy to calculate the reinforcement stresses. First calculate the strain in the bar position (y, z) using the expression for the strain distribution. The bar stress is then calculated using Hooke's law. Thus we get:

$$\sigma_s = E_s [\epsilon_0 + \psi_y(z - z_R) + \psi_z(y - y_R)]$$

In case of initial stresses in the bar we have to add these stresses to the above bending stresses.

3.4.2.2 Time-dependent stresses and strains

An accurate determination of stresses and strains due to time-dependent effects is important in the analysis and design of indeterminate structures, particularly in sway frames or other structures where displacements have great influence on

the solution. In such structures long-time displacements due to shrink and creep can have a major influence and a rigorous method for calculating these effects are therefore of great importance.

Stress-strain relation for concrete at creep

At serviceability limit state it is assumed that a stress increment $\Delta\sigma_c$ introduced at time t_0 and sustained, without change in magnitude, produces at time t a strain given by:

$$\varepsilon_c(t) = \Delta\sigma_c (1 + \varphi) / E_c(t_0)$$

where: $E_c(t_0)$ is the modulus of elasticity of concrete at the instant t_0 ,
 $\varphi [= \varphi(t, t_0)]$ is the creep coefficient.

The value of φ representing the ratio of creep to the instantaneous strains depends on the properties of the concrete and the environment in which it is kept. Values are proposed in various codes but it must be emphasised that these values are rough and may have weaknesses. For a more accurate determination see e.g. [4] chapter 2.3.

When the stress increment $\Delta\sigma_c$ is gradually introduced from zero at time t_0 to its full value at time t the strain at time t is given by:

$$\varepsilon_c(t) = \Delta\sigma_c (1 + \chi_c\varphi) / E_c(t_0)$$

where: $\chi_c [= \chi_c(t, t_0) \cong 0.8]$ is the ageing coefficient of concrete.

The strain may also be written as:

$$\varepsilon_c(t) = \Delta\sigma_c / E_{\text{ceff}}$$

with $E_{\text{ceff}} [E_{\text{ceff}}(t, t_0)]$ is the age-adjusted modulus of elasticity defined by:

$$E_{\text{ceff}} = E_c(t_0) / (1 + \chi_c\varphi)$$

Stress-strain relation for steel at relaxation

At serviceability limit state it is assumed that the stress-strain relation for normal nonprestressed steel is not time dependent, i.e. it will not change during a calculation.

For prestressed steel the stress strain relation is time dependent. The phenomenon, known as relaxation, appears as stress loss at constant strain. The time dependent relation for intrinsic relaxation is usually set to:

$$\sigma_s(t) = \sigma_s(t_0) (1 - \chi_s)$$

Because of changes in steel stress due to loading history (with influences of creep and shrinkage) the intrinsic relaxation have to be reduced in accordance with these stress changes. See e.g. [1] and [4].

Transformed cross sections at creep

For the analysis of stress and strain occurring immediately after load application on reinforced concrete sections, the term *transformed section* is used to represent a section composed of the area of concrete A_c plus the areas of the reinforcement A_s multiplied by $\alpha = E_s / E_c(t_0)$, see ch.3.4.1.3.

When analysis is performed for changes in stress and strain due to creep, shrinkage and relaxation, the term *age-adjusted transformed section* is used. This is composed of the area of concrete A_c plus the areas of the reinforcement A_s multiplied by $\alpha = E_s / E_{\text{ceff}}(t_0)$.

Four analyses steps

Four steps can be followed to determine the stress and strain distributions at time t_0 , immediately after application of section forces, and at time t after occurrence of creep, shrinkage and relaxation.

- **Step 1**
Apply section forces $\{N, M_y, M_z\}^t$ on a transformed section to determine $\{\epsilon_0(t_0), \psi_y(t_0), \psi_z(t_0)\}^t$ which define the instantaneous strain. Multiplication by $E_c(t_0)$ gives $\{\sigma_0(t_0), \gamma_y(t_0), \gamma_z(t_0)\}^t$, which define the instantaneous concrete stresses. Multiplication of the concrete stress by $\alpha(t_0)$ gives the stresses in nonprestressed steel. For the prestressed steel the initial tension must be added.
- **Step 2**
Determine the hypothetical change, in the period t_0 to t , in the strain distribution due to creep and shrinkage if they were free to occur. The change in strain in the reference point R is equal to $[\varphi(t, t_0) \epsilon_0(t_0) + \epsilon_{cs}]$ and the chang-

es in curvature are $[\varphi(t, t_0) \psi_y(t_0) + \Delta e_{csy}]$ and $[\varphi(t, t_0) \psi_z(t_0) + \Delta e_{csz}]$. ϵ_{cs} is the free shrinkage of concrete in the period t_0 to t , and Δe_{csy} and Δe_{csz} are the differential shrinkage in case of not constant shrinkage over the section.

- **Step 3**

Calculate the artificial stress, which when gradually introduced in the concrete during the period t_0 to t , will prevent occurrence of the strain due to free creep and shrinkage calculated in step 2. The restraining stress distribution $\Delta \sigma_{\text{restrained}}$ is given by:

$$\begin{aligned} \left[\Delta \sigma_0 \right] &= \left[\varphi(t, t_0) \epsilon_0(t_0) + \epsilon_{cs} \right] \\ \left\{ \Delta \gamma_y \right\} &= -E_{\text{ceff}} \left\{ \varphi(t, t_0) \psi_y(t_0) + \Delta e_{csy} \right\} \\ \left[\Delta \gamma_z \right]_{\text{restrained}} &= \left[\varphi(t, t_0) \psi_z(t_0) + \Delta e_{csz} \right]_{\text{free}} \end{aligned}$$

- **Step 4**

Determine the section forces, which are the resultants of $\Delta \sigma_{\text{restrained}}$ using section properties for the net concrete section.

The change in concrete strain due to relaxation of prestressed steel can be artificially prevented by the application of, at the level of the prestressed steel, a restraining force equal to $A_s \Delta \sigma_{\text{relax}}$, where $\Delta \sigma_{\text{relax}}$ is the reduced value of the stress relaxation in the period t_0 to t .

Summing up gives $\{\Delta N, \Delta M_y, \Delta M_z\}_{\text{restrained}}^t$, the restraining forces required to artificially prevent the strain change due to shrinkage, creep and relaxation.

To eliminate the artificial restraint apply $\{\Delta N, \Delta M_y, \Delta M_z\}_{\text{restrained}}^t$ in reversed direction on an age-adjusted transformed section and calculate the corresponding changes in strains:

$$\begin{aligned} \left[\Delta \epsilon_0 \right] &= \left[A \ S_y \ S_z \right]^{-1} \left[\Delta N \right] \\ \left\{ \Delta \psi_y \right\} &= (1 / E_{\text{ceff}}) \left[-S_y \ -I_y \ -I_{yz} \right] \left\{ \Delta M_y \right\} \\ \left[\Delta \psi_z \right]_{\text{change}} &= \left[S_z \ I_{yz} \ I_z \right] \left[\Delta M_z \right]_{\text{restrained}} \end{aligned}$$

and stresses:

$$\begin{aligned} & \{ \Delta \sigma_0 \} \quad \{ \Delta \varepsilon_0 \} \\ & \{ \Delta \gamma_y \} = E_{\text{ceff}} \{ \Delta \psi_y \} \\ & \{ \Delta \gamma_z \}_{\text{change}} \quad \{ \Delta \psi_z \}_{\text{change}} \end{aligned}$$

Final stresses and strains

The concrete strain distribution at time **t** is the sum of the elastic strain (**step 1**) and the change in strain due to shrinkage, creep and relaxation (**step 4**).

$$\begin{aligned} & \{ \varepsilon_0 \} \quad \{ \varepsilon_0 \} \quad \{ \Delta \varepsilon_0 \} \\ & \{ \psi_y \} = \{ \psi_y \} + \{ \Delta \psi_y \} \\ & \{ \psi_z \}_{\text{sum}} \quad \{ \psi_z \}_{\text{el}} \quad \{ \Delta \psi_z \}_{\text{change}} \end{aligned}$$

The concrete stress distribution at time **t** is the sum of the elastic stress (**step 1**), the restraint stresses (**step 3**) and the changes in stresses due to the reversed restraint (**step 4**).

$$\begin{aligned} & \{ \sigma_0 \} \quad \{ \sigma_0 \} \quad \{ \Delta \sigma_0 \} \quad \{ \Delta \sigma_0 \} \\ & \{ \gamma_y \} = \{ \gamma_y \} + \{ \Delta \gamma_y \} + \{ \Delta \gamma_y \} \\ & \{ \gamma_z \}_{\text{sum}} \quad \{ \gamma_z \}_{\text{el}} \quad \{ \Delta \gamma_z \}_{\text{restrained}} \quad \{ \Delta \gamma_z \}_{\text{change}} \end{aligned}$$

The strain ε in reinforcement at the time **t** is the sum of the elastic strains (**step 1**) and the change in strain due to the reversed restraint (**step 4**). The final stress is then

simply calculated as $E_s \varepsilon$. In prestressed reinforcement the initial stresses and the reduced relaxation must be added to get the final.

3.4.2.3 Stresses and strain in cracked sections

Stresses and strain in a cracked section can generally not be solved by a direct method because only the compressed part of the section is active and the part in tension is ignored. The properties for the active part are then dependent of the position of the neutral axis and thus this must be determined before calculation of stresses and strains may be completed. On the other hand the position of the neutral axis depends on the stress-strain distribution, so obviously an iterative method is needed.

Obviously a stress distribution must be determined that satisfies the equilibrium of the cracked section. As before (see ch. 3.4.2) we get:

$\{N\} = \left\{ \int \sigma \, dA \right\}$ The stress at any fiber is a function of strain in accordance with the stress-strain relations, which generically may be non-linear. However, in serviceability limit state these relations are often considered to be linear in accordance with Hooke's law.

$\{M_y\} = \left\{ -\int \sigma \, z \, dA \right\}$

$\{M_z\} = \left\{ \int \sigma \, y \, dA \right\}$

At determination of the position of the neutral axis position we assume as a first attempt uncracked section:

$$\begin{bmatrix} \varepsilon_0 \\ \psi_y \\ \psi_z \end{bmatrix}_{\text{trial 1}} = \begin{bmatrix} A & S_y & S_z \\ -S_y & -I_y & -I_{yz} \\ S_z & I_{yz} & I_z \end{bmatrix}^{-1} \begin{bmatrix} N \\ M_y \\ M_z \end{bmatrix}_{\text{section}}$$

where the section properties are properties of a transformed uncracked section.

By numerical integration, using first trial values of the strain parameters and the stress-strain relation, determine $\{N, M_y, M_z\}_{\text{calculated}}$ ignoring concrete in tension.

Determine a vector of residuals:

$$\begin{bmatrix} R_1 \\ R_2 \\ R_3 \end{bmatrix} = \begin{bmatrix} N \\ M_y \\ M_z \end{bmatrix}_{\text{calculated}} - \begin{bmatrix} N \\ M_y \\ M_z \end{bmatrix}_{\text{section}}$$

Use Newton-Raphson iteration to bring $\{R\}$ close to $\{0\}$:

$$\begin{bmatrix} \partial R_1 / \partial \varepsilon_0 & \partial R_1 / \partial \psi_y & \partial R_1 / \partial \psi_z \\ \partial R_2 / \partial \varepsilon_0 & \partial R_2 / \partial \psi_y & \partial R_2 / \partial \psi_z \\ \partial R_3 / \partial \varepsilon_0 & \partial R_3 / \partial \psi_y & \partial R_3 / \partial \psi_z \end{bmatrix} \begin{bmatrix} \Delta \varepsilon_0 \\ \Delta \psi_y \\ \Delta \psi_z \end{bmatrix}_1 = - \begin{bmatrix} R_1 \\ R_2 \\ R_3 \end{bmatrix}$$

Solution of this equation gives incremental strain parameters leading to improved trial values:

$$\begin{aligned} \{\varepsilon_0\} & \quad \{\varepsilon_0\} & \quad \{\Delta\varepsilon_0\} \\ \{\psi_y\} & = \{\psi_y\} & + \{\Delta\psi_y\} \\ \{\psi_z\}_{\text{trial } i} & \quad \{\psi_z\}_{\text{trial } (i-1)} & \quad \{\Delta\psi_z\}_i \end{aligned}$$

The elements of the partial derivative matrix can be taken equal to $\mathbf{E}_c(\mathbf{t})$ multiplied by the area properties of a transformed section in the cracked:

$$\begin{aligned} \begin{bmatrix} \partial R_1 / \partial \varepsilon_0 & \partial R_1 / \partial \psi_y & \partial R_1 / \partial \psi_z \\ \partial R_2 / \partial \varepsilon_0 & \partial R_2 / \partial \psi_y & \partial R_2 / \partial \psi_z \\ \partial R_3 / \partial \varepsilon_0 & \partial R_3 / \partial \psi_y & \partial R_3 / \partial \psi_z \end{bmatrix} & \quad \begin{bmatrix} A & S_y & S_z \\ -S_y & -I_y & -I_{yz} \\ S_z & I_{yz} & I_z \end{bmatrix} \\ \approx E_c(\mathbf{t}) & \end{aligned}$$

For any trial values of the strain parameters $\{\varepsilon_0, \psi_y, \psi_z\}^t$ the equation of the neutral axis is:

$$e_0 + \psi_y(z - z_R) + \psi_z(y - y_R) = 0$$

This line will indicate the compression zone to be included in calculating the transformed cracked section above. The partial derivative matrix determined by the trial values $\{\varepsilon_0, \psi_y, \psi_z\}_{\text{trial } 1}$ may be used in all iterations, but recalculation in every step can speed up the convergence.

Time-dependent stresses in cracked sections

Calculating time-dependent stresses and strains in cracked sections is performed principally as in ch. 3.4.2.2. However, as a result of the change of stress distribution between concrete and reinforcement the neutral axis changes position. In cracked sections this calls for a minor iteration to determine the final position of the neutral axis after time-dependent changes in stress-strain distributions.

3.4.2.4 Cracking

Cracking stress

A section is assumed to be in cracked state when the maximum tensile stress, calculated on basis of an uncracked section, exceeds the tensile strength of the concrete. At bending the crack condition is then expressed as:

$$\sigma_n + \sigma_m \leq f_{ctm}$$

where:

σ_n is stress of normal force,

σ_m is stress of moment,

f_{ctm} is the mean tensile design strength.

Cracking moment

Cracking moment is the moment that barely causes cracking. With section forces N and M with belonging stresses σ_n and σ_m we find the factor c that induces cracking at forces N and $c M$ by:

$$c = (f_{ctd} - \sigma_n) / \sigma_m$$

and cracking moment by:

$$M_{crack} = c M$$

Crack width calculation

The crack width w_k may be calculated from:

$$w_k = S_{r,max} (\epsilon_{sm} - \epsilon_{cm})$$

where:

$S_{r,max}$ is the maximum crack spacing,

ϵ_{sm} is the mean strain in the reinforcement,

ϵ_{cm} is the mean strain in the concrete between cracks.

$\epsilon_{sm} - \epsilon_{cm}$ may be calculated from:

$$\epsilon_{sm} - \epsilon_{cm} = [\sigma_s - k_t f_{ct,eff} / \rho_{p,eff} (1 + \alpha_e \rho_{p,eff})] / E_s \geq 0,6 \sigma_s / E_s$$

where:

σ_s is the stress in the tension reinforcement assuming cracked section,

α_e is the ratio E_s / E_{cm}

$f_{ct,eff}$ is the mean value of the tensile strength of the concrete when the first crack occur,

$$f_{ct,eff} = f_{ctm}$$

$$\rho_{p,\text{eff}} = (A_s + \zeta_1^2 A_p) / A_{c,\text{eff}}$$

A_p is the area of pre or post-tensioned tendons within $A_{c,\text{eff}}$,

$A_{c,\text{eff}}$ is the effective area of concrete as calculated below,

ξ_1 is the adjusted ratio of bond strength as calculated below,

k_t is a factor dependent on the duration of the load,

$k_t = 0,6$ for short term loading and $0,4$ for long term loading.

Effective area of $A_{c,\text{eff}}$

$A_{c,\text{eff}}$ is the effective area of concrete of depth $h_{c,\text{eff}}$ where $h_{c,\text{eff}}$ is the lesser of:

$2,5 (h - d)$, $(h - x) / 3$ or $h / 2$, see figure below:

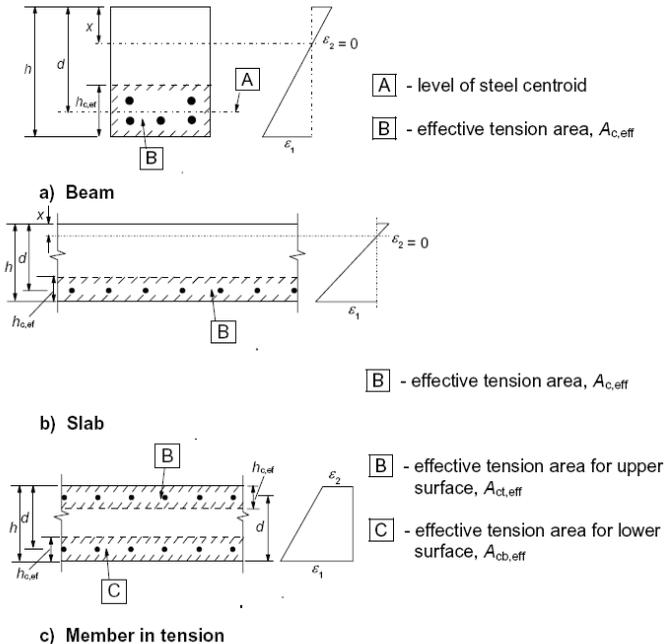


Figure 7.1: Effective tension area (typical cases)

Adjusted ratio of bond strength ξ_1

ξ_1 is the adjusted ratio of bond strength taking into account the different diameters of prestressing and reinforcing steel:

$$\xi_1 = (\xi \phi_s / \phi_p)^{0,5}$$

where:

ξ is the ratio of bond strength according to the below table,

ϕ_s is the largest bar diameter,

ϕ_p is equivalent diameter of prestressing steel.

Table 6.2: Ratio of bond strength, ξ , between tendons and reinforcing steel

| prestressing steel | ξ | | |
|-----------------------|----------------|------------------------|---------------|
| | pre-tensioned | bonded, post-tensioned | |
| | | $\leq C50/60$ | $\geq C70/85$ |
| smooth bars and wires | Not applicable | 0,3 | 0,15 |
| strands | 0,6 | 0,5 | 0,25 |
| indented wires | 0,7 | 0,6 | 0,3 |
| ribbed bars | 0,8 | 0,7 | 0,35 |

Note: For intermediate values between C50/60 and C70/85 interpolation may be used.

Crack spacing $S_{r,max}$

For bonded reinforcement with spacing $\leq 5(c + \phi / 2)$ the crack spacing is calculated as:

$$S_{r,max} = k_3 c + k_1 k_2 k_4 \phi / \rho_{p,eff}$$

where:

ϕ is the bar diameter in mm. If more than one bar size is present an average bar size ϕ_{eq} should be used,

$$\phi_{eq} = (n_1 \phi_1^2 + n_2 \phi_2^2) / (n_1 \phi_1 + n_2 \phi_2)$$

c is the cover to the longitudinal reinforcement,

$k_1 = 0,8$ for high bond bars, $1,6$ for plain bars (e.g. prestressing tendons),

$k_2 = 0,5$ for bending, $1,0$ for pure tension,

$$k_3 = 3,4$$

$$k_4 = 0,425$$

$\rho_{p,eff}$ as above.

For not bonded reinforcement or reinforcement with spacing $> 5 (c + \phi / 2)$ the crack spacing is calculated as:

$$S_{r,max} = 1,3 (h - x)$$

where:

x is the neutral axis depth.

The crack spacing should be calculated in the direction of the principle tensile stress as:

$$S_{r,max} = 1 / (\cos \theta / S_{r,max,y} + \sin \theta / S_{r,max,z})$$

where:

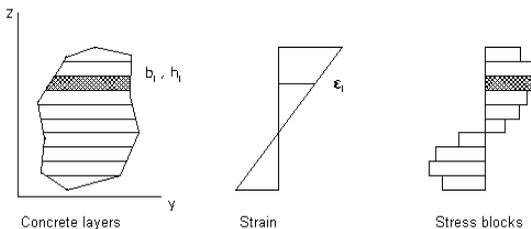
θ is the angle between the reinforcement in the y-direction and the direction of the principal tensile stress,

$S_{r,max,y}$ and $S_{r,max,z}$ are the crack spacings calculated in the y and z directions respectively.

3.4.3 Stresses and strains at non-linear analysis

Non-linear section analysis is used when stresses or strains are so large that the relation between stress and strain no longer is assumed to be linear. This is generally the case at Ultimate Limit State.

3.4.3.1 Internal section forces from stresses and strain



At nonlinear section analysis it is assumed that (see ch. 3.4.2):

- a) Plane cross-sections remain plane at bending (Bernoulli's hypothesis).
- b) Only normal stresses occur in pure bending (Navier's hypothesis).

In order to determine resultants to stresses in the section, generally numerical integration is needed. A simple method to make the integration is to divide the section into a number of concrete layers and reinforcement bars and then analyze each layer and bar separately. Strain and stress are determined in the mid-depth of each component and a numerical integration of the resulting moments and forces is performed:

$$N = \sum_{i=1}^n f_{ci} b_i h_i + \sum_{j=1}^m (f_{sj} - f_{cj}) A_{sj}$$

$$M = \sum_{i=1}^n -f_{ci} b_i h_i (z_{cg} - z_{ci}) + \sum_{j=1}^m (f_{sj} - f_{cj}) A_{sj} (z_{cg} - z_j)$$

with stresses according to:

$f_{ci} = f_c(\epsilon_{ci})$ where ϵ_{ci} is concrete strain at layer position,

$f_{cj} = f_c(\epsilon_{cj})$ where ϵ_{cj} is concrete strain at bar position,

$f_{sj} = f_s(\epsilon_{sj})$ where ϵ_{sj} is steel strain at bar position,

where: **n, m** = number of concrete layers and reinforcing bar elements,

b_i, h_i = width and depth of a concrete layer **i**,

A_s = cross-sectional area of reinforcing bar,

z_{cg} = **z**-coordinate for centroid of section in local coordinate system with **y**-axis parallel to neutral axis,

f_c = stresses in concrete,

f_s = stresses in reinforcement,

f_c(ε**)** = stress-strain relation (non-linear) for concrete,

f_s(ε**)** = stress-strain relation (non-linear) for steel.

If the section is divided into a sufficient number of layers the accuracy will be satisfactory even if large non-linearity in the stress distribution exists.

3.4.3.2 Stress-strain relation for concrete

A rectangular stress distribution as shown below with height λx may be assumed.

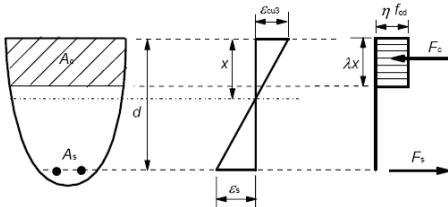


Figure 3.5: Rectangular stress distribution

$$\lambda = 0,8 \quad \text{for } f_{ck} \leq 50 \text{ MPa} \quad (3.19)$$

$$\lambda = 0,8 - (f_{ck} - 50)/400 \quad \text{for } 50 < f_{ck} \leq 90 \text{ MPa} \quad (3.20)$$

and

$$\eta = 1,0 \quad \text{for } f_{ck} \leq 50 \text{ MPa} \quad (3.21)$$

$$\eta = 1,0 - (f_{ck} - 50)/200 \quad \text{for } 50 < f_{ck} \leq 90 \text{ MPa} \quad (3.22)$$

Note: If the width of the compression zone decreases in the direction of the extreme compression fibre, the value ηf_{cd} should be reduced by 10%.

The value of the design compressive strength is defined as:

$$f_{cd} = \alpha_{cc} f_{ck} / \gamma_c$$

where:

$$\gamma_{cc} = 1,0,$$

γ_c is the partial safety factor as above.

The value of the design tensile strength is defined as:

$$f_{ctd} = \alpha_{ct} f_{ctk,0.05} / \gamma_c$$

where:

$$\alpha_{ct} = 1,0,$$

γ_c is the partial safety factor as above.

NA UK

$$\alpha_{cc} = 0,85$$

NA Finland

$$\alpha_{cc} = 0,85$$

3.4.3.3 Stress-strain relation for reinforcement steel

Strain limit $\epsilon_{ud} = 0,9 \epsilon_{uk}$

NA Denmark

$$\epsilon_{ud} = f_{yd} / E_s$$

NA Finland

$$\epsilon_{ud} = 0,02$$

3.4.3.4 Calculation of stress and strains due to section forces

At non-linear section analysis determination of the strain distribution on a section including position of the neutral axis is generally not possible to perform with a direct method. Instead an iterative method must be. The method that is described in ch. 3.4.2.3 for cracked section is suitable to use at non-linear calculation too. The calculation of section forces out of assumed strains is then performed using the method described in ch. 3.4.3.1.

With this method primarily the strain distribution is determined and after that the stresses are calculated using the non-linear stress-strain relation.

3.4.4 Elongation and curvature

Elongation and curvature of a section is immediately available when the strain distribution has been calculated (linear or non-linear) as the components of the strain distribution represent the strain ϵ_0 in reference point **R** and the curvatures ψ_y and ψ_z .

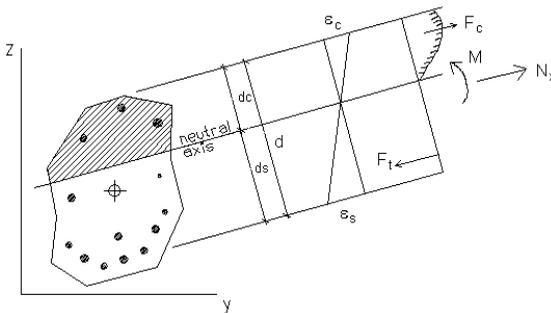
In structural calculations these values may be used directly for an uncracked section. For a cracked section it is unfavourable to use the cracked values directly as influence of uncracked concrete between cracks is not regarded. This influence is taken care of by the distribution coefficient ξ (see [7] ch. 7.4.3), which

approximately regards the distribution between cracked and uncracked part of the element. We get:

$$\begin{Bmatrix} \varepsilon_0 \\ \psi_y \\ \psi_z \end{Bmatrix}_{\text{averagh}} = \zeta \begin{Bmatrix} \varepsilon_0 \\ \psi_y \\ \psi_z \end{Bmatrix}_{\text{cr}} + (1 - \zeta) \begin{Bmatrix} \varepsilon_0 \\ \psi_y \\ \psi_z \end{Bmatrix}_{\text{oner}}$$

The coefficient ζ is generally calculated in connection to cracking (see ch. 3.4.2.4).

3.4.5 Ultimate moment capacity



Moment capacity in ultimate limit state is most easily calculated with a fixed neutral axis inclination, e.g. parallel to either **y** or **z**-axis. As non-linear material and cracking are assumed iteration is required to establish equilibrium together with external normal force

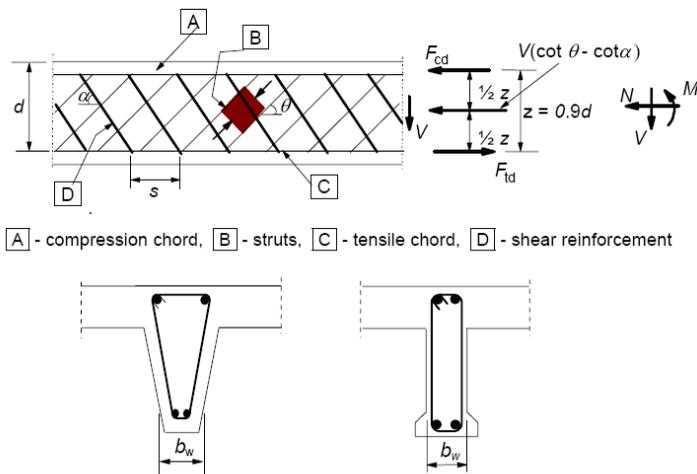
and at the same time calculate current moment capacity.

Start the calculation by setting maximum allowed strain in extreme concrete fiber (ε_{cmax}) and extreme reinforcement bar (ε_{smax}) and calculate force resultants in compression zone and tension zone in order to determine if compression or tension is decisive. Iteration continues with adjustment of neutral axis position to get maximum utilization of decisive materials at established equilibrium conditions.

Moment capacity may also be calculated in an optional direction, either as a fixed neutral axis inclination (other than **y** or **z**-axis) or as a fixed moment vector inclination. In the latter case an iteration is generally demanded to establish the neutral axis direction that corresponds to the given moment vector inclination.

3.4.6 Shear capacity with shear reinforcement

The design of members with shear reinforcement is based on a truss model (see figure below):



A - compression chord, B - struts, C - tensile chord, D - shear reinforcement

Figure 6.5: Truss model and notation for shear reinforced members

where:

α is the angle between shear reinforcement and the beam axis,

θ is the angle between the concrete compression strut and the beam axis perpendicular to the shear force,

F_{td} is the design value of the tensile force in the longitudinal reinforcement,

F_{cd} is the design value of the compression force in the direction of the longitudinal member axis,

b_w is the minimum width between tension and compression chords,

z is the inner lever arm. In shear analysis without axial force, the approximate value $z = 0.9 d$ may normally be used.

z is the inner lever arm. In shear analysis without axial force, the approximate value $z = 0,9 d$ may normally be used.

3.4.7 Shear capacity of the shear reinforcement

The shear capacity of a section with shear reinforcement is considered to be sufficient if:

$$V_{Ed} \leq V_{Rd,s} \quad \text{but} \quad V_{Ed} \leq V_{Rd,max}$$

where:

$V_{Rd,s}$ is the shear capacity of a section with shear reinforcement,

$V_{Rd,max}$ is the max capacity.

$$V_{Rd,s} = A_{sw} / s \cdot z \cdot f_{ywd} (\cot \theta + \cot \alpha) \sin \alpha \text{ or}$$

$$s = [A_{sw} \cdot z \cdot f_{ywd} (\cot \theta + \cot \alpha) \sin \alpha] / V_{Ed}$$

$$V_{Rd,max} = \alpha_{cw} \cdot b_w \cdot z \cdot v_1 \cdot f_{cd} (\cot \theta + \cot \alpha) / (1 + \cot^2 \theta)$$

where:

f_{ywd} is the design yield strength of the shear reinforcement,

A_{sw} is the area of the shear reinforcement,

s is the spacing of the stirrups,

The angle θ should be limited to $1 \leq \cot \theta \leq 2,5$

v_1 is a strength reduction factor for concrete cracked in shear.

If the design stress of the shear reinforcement is below 80% of the yield stress f_{yk} then:

$$v_1 = 0,6 \quad \text{for} \quad f_{ck} \leq 60 \text{ MPa}$$

$$v_1 = 0,91 - f_{ck} / 200 > 0,5 \quad \text{for} \quad f_{ck} > 60 \text{ MPa}$$

In this case $f_{ywd} = 0,8 f_{ywk}$

else:

$$v_1 = 0,6 (1 - f_{ck} / 250) \quad (f_{ck} \text{ in MPa})$$

α_{cw} is a coefficient taking account of the state of the stress in the compression chord,

$\alpha_{cw} = 1,0$ for non-pre stressed structures.

The maximum effective shear reinforcement $A_{sw,max}$ for $\cot \theta = 1$ follows from:

$$A_{sw,max} f_{ywd} / (b_w s) \leq 0,5 \alpha_{cw} v_1 f_{cd} / \sin \alpha$$

In regions where there is no discontinuity of V_{Ed} (e.g. uniformly distributed loading) the shear reinforcement in any length increment $l = z (\cot \theta + \cot \alpha)$ may be calculated using the smallest value of V_{Ed} in the increment.

NA British

If shear co-exists with applied tension then $\cot \theta$ should be taken as 1,0.

If the design stress of the shear reinforcement is below 80% of the yield stress f_{yk} then:

$$v_1 = 0,54 (1 - 0,5 \cos \alpha) \text{ for } f_{ck} \leq 60 \text{ MPa}$$

$$v_1 = (0,84 - f_{ck} / 200) (1 - 0,5 \cos \alpha) > 0,5 \text{ for } f_{ck} > 60 \text{ MPa}$$

NA Denmark

If reinforcement class B or C is used then the inclination θ of the compressive stress should be chosen as:

$$\tan \alpha / 2 \leq \cot \theta \leq 2,5$$

where curtailed reinforcement is chosen:

$$\tan \alpha / 2 \leq \cot \theta \leq 2,0$$

where:

α is the angle between shear reinforcement and the beam axis.

If reinforcement class A is used then $\cot \theta = 1,0$

The strength reduction factor for concrete cracked in shear is calculated as:

$$v_1 = 0,76 (1 - f_{ck} / 200) \quad (f_{ck} \text{ in MPa}) \quad (5.103NA)$$

3.4.8 Shear capacity without shear reinforcement

The shear capacity of a section without shear reinforcement is considered to be sufficient if:

$$V_{Ed} \leq V_{Rd,c}$$

where:

V_{Ed} is the shear design force supplied by analysis,

$V_{Rd,c}$ is the shear capacity of the concrete.

However minimum shear reinforcement should nevertheless be provided except for slabs where transverse redistribution of loads is possible.

3.4.9 Shear capacity of the concrete

$$V_{Rd,c} = [C_{Rd,c} k (100 \rho_1 f_{ck})^{1/3} + k_1 \sigma_{cp}] b_w d$$

with a minimum of:

$$V_{Rd,c} = [v_{min} + k_1 \sigma_{cp}] b_w d$$

where:

$$C_{Rd,c} = 0,18 / \gamma_c$$

$$k_1 = 0,15$$

f_{ck} is in MPa

$$k = 1 + (200 / d)^{0,5} \leq 2,0, \text{ with } d \text{ in mm}$$

$$\rho_1 = A_{st} / (b_w d)$$

A_{st} is the area of the tensile reinforcement, which extends $\geq (l_{bd} + d)$ beyond the section considered (see figure below),

b_w is the smallest width of the cross-section in the tensile area (mm).

$$\sigma_{cp} = N_{Ed} / A_c < 0,2 \text{ (MPa)}$$

where:

N_{Ed} is the axial force in the cross-section due to loading or pre stressing (in N) ($N_{Ed} > 0$ for compression). The influence of imposed deformations on N_E may be ignored.

A_c is the area of concrete cross section (mm^2),

$V_{Rd,c}$ is in (N)

$$v_{\min} = 0,035 k^{3/2} f_{ck}^{1/2}$$

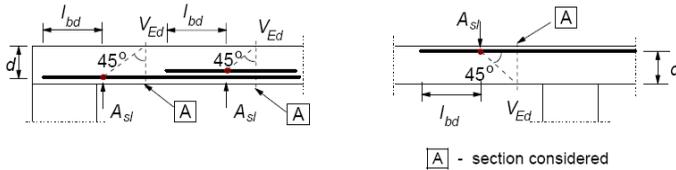


Figure 6.3: Definition of A_{sl}

The shear force V_{Ed} should always satisfy the condition:

$$V_{Ed} \leq 0,5 b_w d v f_{cd}$$

where:

$$0,6 (1 - f_{ck} / 250) \quad (f_{ck} \text{ in MPa})$$

3.4.10 Rules for shear reinforcement

The shear reinforcement should form an angle α between 45 and 90 degrees to the longitudinal axis of the structural element.

The ratio of shear reinforcement is given by:

$$\rho_w = A_{sw} / (s b_w \sin \alpha)$$

where:

A_{sw} is the area of shear reinforcement within length s ,

The minimum shear reinforcement ratio is given by:

$$\rho_{w,\min} = (0,08 f_{ck}^{0,5}) / f_{yk}$$

The maximum spacing between shear assemblies should not exceed $s_{l,max}$:

$$s_{l,max} = 0,75 d (1 + \cot \alpha)$$

The transverse spacing of the legs in a series of shear links should not exceed $s_{t,max}$:

$$s_{t,max} = 0,75 d \leq 600 \text{ mm}$$

NA Denmark

The minimum shear reinforcement ratio is given by:

$$\rho_{w,min} = (0,063 f_{ck}^{0,5}) / f_{yk}$$

4 References

1. *Euro code*, EN 1992-1-1, 2004.
2. *Handboken Bygg, Konstruktion*, Avsnitt K17-K20, Stockholm, 1985.
3. *Handboken Bygg, Allmänna grunder*, Avsnitt A30, Stockholm, 1985.
4. *Betonghandbok - Konstruktion* utgåva 2, K. Cederwall, M Lorentsen, L. Östlund m fl, Svensk Byggtjänst, 1990.
5. *Time-Dependant Stresses in Prestressed Concrete Sections of General Shape*, M. Kawakami, A. Ghali, PCI Journal May-June 1996.
6. *Cracking, Ultimate Strength and Deformations of Prestressed Concrete Sections of General Shape*, M. Kawakami, A. Ghali, PCI Journal May-June 1996.
7. *Nonlinear Analysis of Reinforced Concrete Frames Subjected to Thermal and Mechanical Loads*, Frank J. Vecchio, ACI Structural Journal /Nov-Dec 1987.